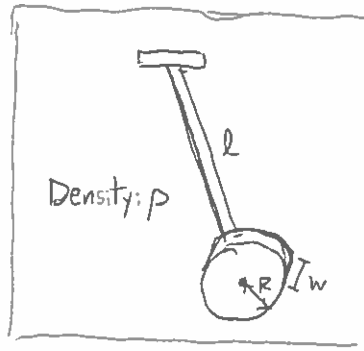
Pendulum Bob Moment

(solution)



- By the parallel-axis theorem, $\boxed{I = I_{CM} + Md^2}$
- For a cylinder, $I_{CM} = \int_0^{2\pi}\int_0^{w}\int_0^{R} \rho r^2 \cdot \underbrace{r\,dr\,dw\,d\phi}_{\text{measure of integration}} = (2\pi)w\rho\cdot\frac{1}{4}R^4$

  or $\boxed{I_{CM} = \frac{\pi}{2}\rho w R^4}$
- And $M = \rho\cdot[\text{Vol.}] = \boxed{\rho\pi R^2 w}$, with $\boxed{d = \ell + R}$
- Putting this all together:

$$I = \frac{\pi}{2}\rho w R^4 + \pi\rho w(\ell+R)^2 R^2$$
$$= \pi\rho w\left[\tfrac{1}{2}R^4 + R^2(R^2 + 2R\ell + \ell^2 R^2)\right]$$
$$= \boxed{\pi\rho w\left[\tfrac{3}{2}R^4 + 2\ell R^3 + \ell^2 R^2\right]} \approx 637\ \text{Kgm}^2$$

Plugging in mean values

- Partial derivatives:

Plugging in mean values...

$$\left(\frac{\partial I}{\partial \rho}\right) = \pi w\left[\tfrac{3}{2}R^4 + 2\ell R^3 + \ell^2 R^2\right] = \frac{I}{\rho} \longrightarrow 0.0720\ m^5$$

$$\left(\frac{\partial I}{\partial w}\right) = \frac{I}{w} \longrightarrow 5.31\cdot 10^4\ \text{Kg}\cdot m$$

$$\left(\frac{\partial I}{\partial \ell}\right) = \pi\rho w\left[2R^3 + 2\ell R^2\right] \longrightarrow 701\ \text{kg}\cdot m$$

$$\left(\frac{\partial I}{\partial R}\right) = \pi\rho w\left[6R^3 + 6\ell R^2 + 2\ell^2 R\right] \longrightarrow 2.45\cdot 10^3\ \text{kg}\cdot m$$

- The overall uncertainty is:

$$\Delta I = \sqrt{\left(\frac{\partial I}{\partial \rho}\right)^2(\Delta\rho)^2 + \left(\frac{\partial I}{\partial w}\right)^2(\Delta w)^2 + \left(\frac{\partial I}{\partial L}\right)^2(\Delta L)^2 + \left(\frac{\partial I}{\partial R}\right)^2(\Delta R)^2} \approx 32.1$$

.. so I'd quote this result as $\boxed{I = 637\ \text{Kg}\cdot m^2 \pm 32\ \text{Kg}\cdot m^2}$