Problem 15.7

Young and Freedman 14th Edition.

Given:

Transverse waves on a string have a wave speed of 8.00 m/s, amplitude 0.0700 m, and wavelength 0.320 m. The waves travel in the -x direction, and at t = 0, the x=0 end of the string has its maximum upward displacement.

In[26]:=
```
v = 8.00; A = 0.0700; λ = 0.320;
```

(a) Compute related quantities f, T, k

In[27]:=
```
T = λ / v
```
Out[27]=
```
0.04
```

In[28]:=
```
f = 1 / T
```
Out[28]=
```
25.
```

In[29]:=
```
ω = 2 π / T
```
Out[29]=
```
157.08
```

In[30]:=
```
k = 2 π / λ
```
Out[30]=
```
19.635
```

(b) Write out a function for the wave y(x, t)

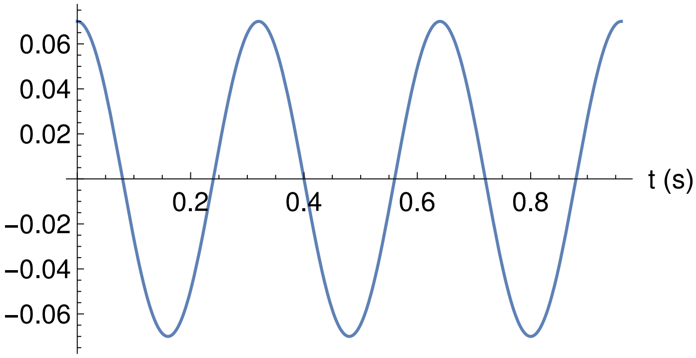
The equation for the traveling wave is the following. The '+' sign is because the wave travels to the left. The Cos[ ] is because we are given the initial condition y[0, 0] = A.

In[31]:=
```
y[x_, t_] := A Cos[k x + ω t]
```

Here is a snapshot of the wave at time t = 0.

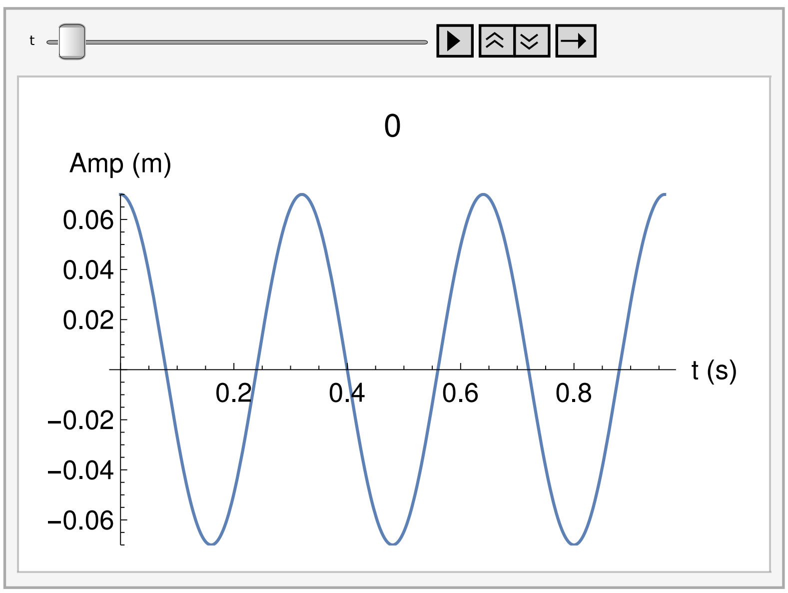
In[32]:=
```
Plot[y[x, 0], {x, 0, 3 λ},
 AxesLabel → {"t (s)", "Amp (m)"}, LabelStyle → Larger]
```

Out[32]=



In[33]:=
```
Animate[Plot[y[x, t], {x, 0, 3 λ}, PlotRange → {-A, A}, PlotLabel → t,
  AxesLabel → {"t (s)", "Amp (m)"}, LabelStyle → Larger],
 {t, 0, 10 T, 0.001, AnimationRate → 0.01, AnimationRunning → False}]
```

Out[33]=



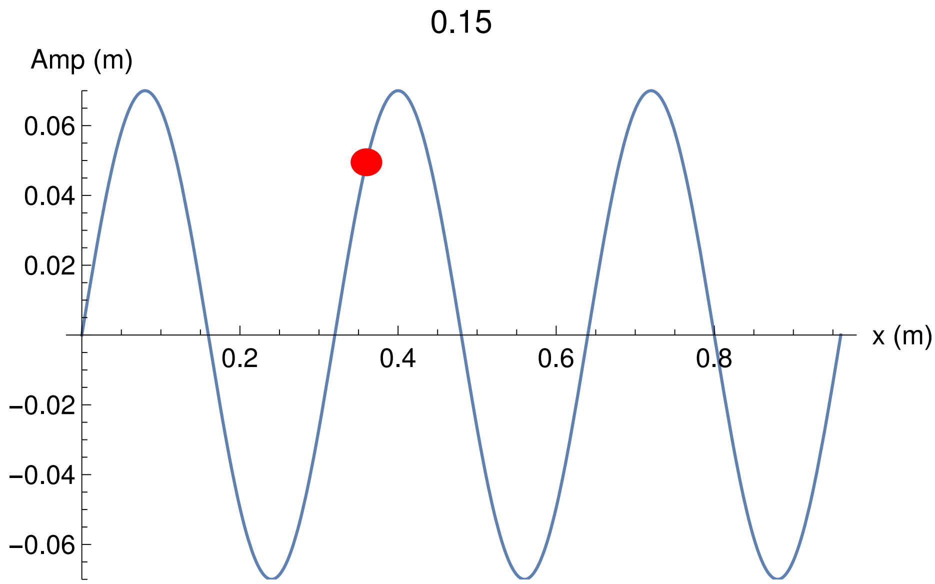
(c). Find the transverse displacement of a particle at x = 0.360, t = 0.150.

In[34]:=
```
xc = 0.360; tc = 0.150;
```

In[35]:=
```
y[xc, tc]
```

Out[35]= 0.0494975

In[36]:=
```
Show[{Plot[y[x, tc], {x, 0, 3 λ}, PlotRange → {-A, A}, PlotLabel → tc],
   Graphics[{Red, Disk[{xc, y[xc, tc]}, {0.02, 0.004}]}]},
  AxesLabel → {"x (m)", "Amp (m)"}, LabelStyle → Larger, ImageSize → Scaled[0.75]
]
```

Out[36]=


(d). Look at that same x-position, namely $x_c = 0.360$. When does it next reach the maximum?

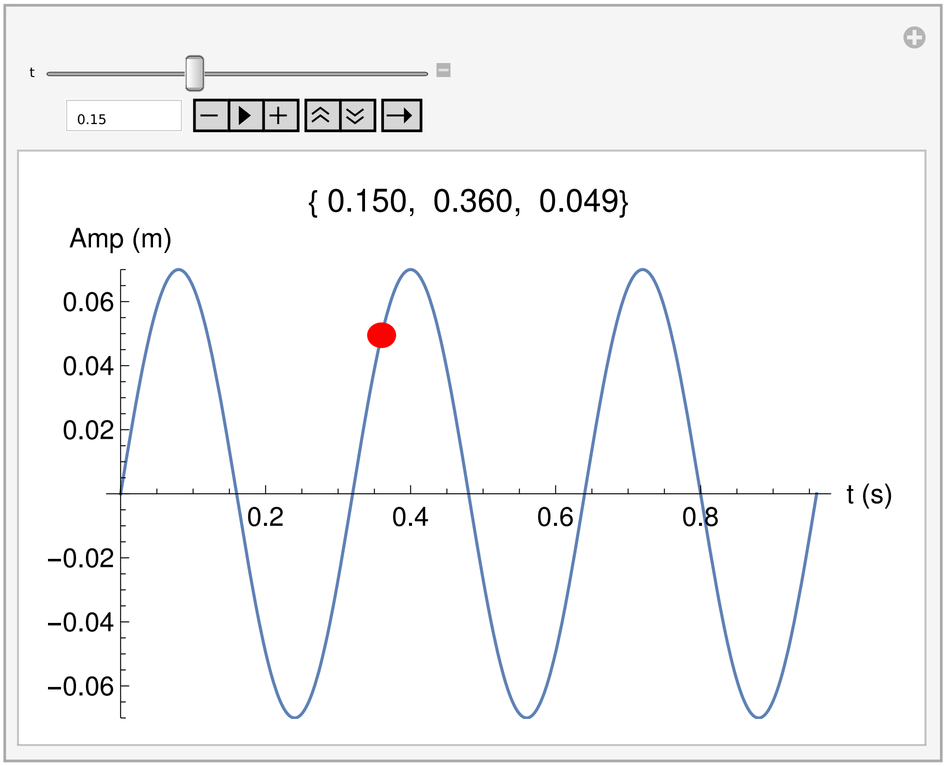
There are two approaches to take. The first is essentially graphical -- draw graphs of the function and look at the clock, waiting for the height at x_c to reach a maximum. The second is a more analytical approach.

Graphical and numerical approach

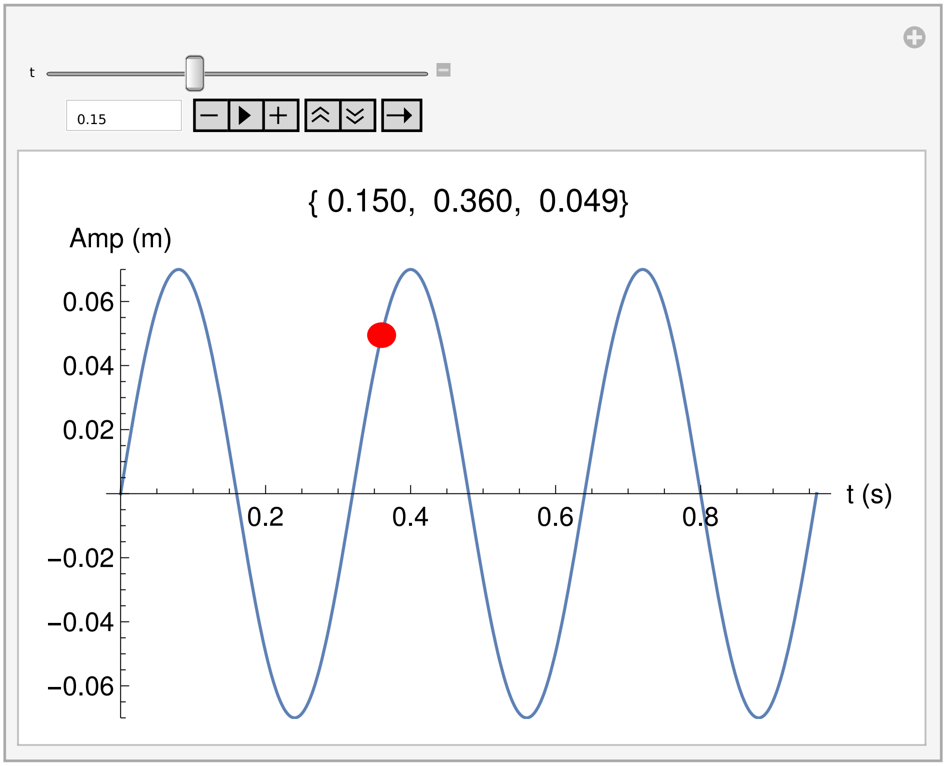
In[37]:=
```
nf[x_] := PaddedForm[Chop[x], {4, 3}](* Handy formatting function *)
```

In[38]:=
```
Manipulate[Show[{Plot[y[x, t], {x, 0, 3 λ},
    PlotRange → {-A, A}, PlotLabel → {nf[t], nf[xc], nf[y[xc, t]]}],
   Graphics[{Red, Disk[{xc, y[xc, t]}, {0.02, 0.004}]}]},
  AxesLabel → {"t (s)", "Amp (m)"}, LabelStyle → Larger, ImageSize → Scaled[0.75]
 ], {{t, 0.15}, 0, 10 T, 0.001, Appearance → "Open", AnimationRate → 0.01}]
```

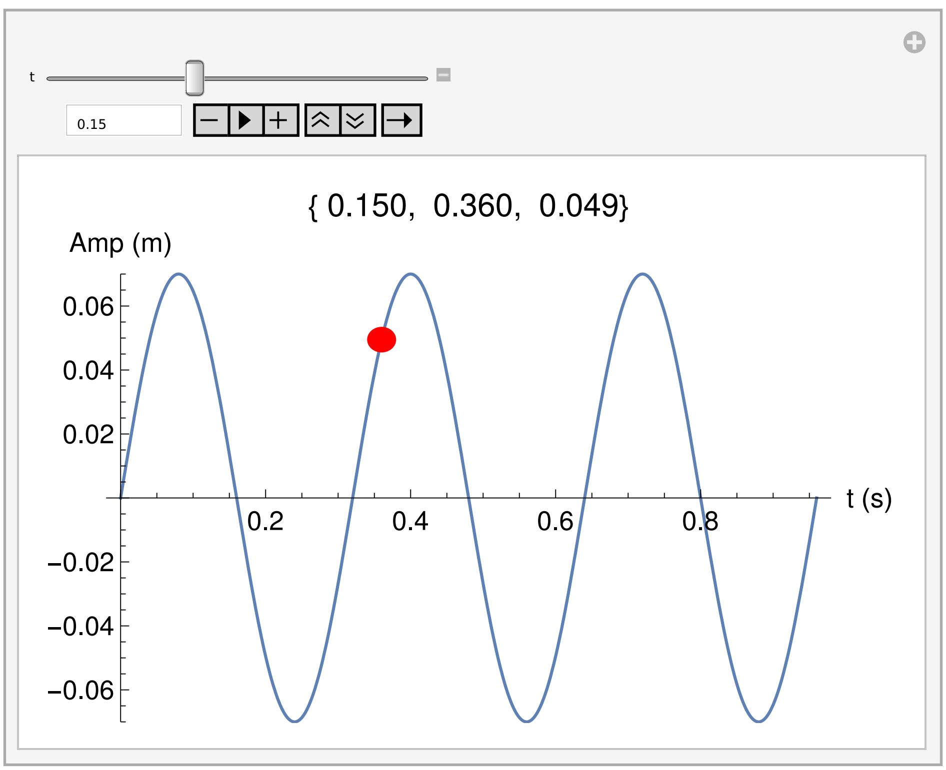
Out[38]=



In[39]:=



Out[39]=



There are many times at which y[xc] will be the maximum. You need to tell *Mathematica* where to start looking. From the animation, it appears to be about 0.155 s.

In[40]:=
```
FindRoot[y[xc, t] == A, {t, tc}]
```
Out[40]=
```
{t → 0.155}
```

The difference from part (c) is thus about 0.005 s.

In[41]:=
```
t - tc /. %
```
Out[41]=
```
0.00499999
```

The next time y[xc] is a maximum is at t = 0.195 s. (Tell Mathematica to start looking at 0.18 s, that is, at some time significantly later than 0.155 s.)

In[52]:=
```
FindRoot[y[xc, t] == A, {t, 0.18}]
```
Out[52]=
```
{t → 0.195}
```

Compute how much later that is than the previous maximum. It should be one period.

In[53]:=
```
Δt = 0.195 - 0.155
```
Out[53]=
```
0.04
```

In[54]:=
```
T == Δt
```
Out[54]=
```
True
```

Analytic approach

The problem is to solve for the time when $y(x_c,\ t) = A$, that is, the maximum. The position is the same as in part (c), but the time is unknown.

$$y(x_c,\ t) = A\cos(k\,x_c + \omega\,t)$$
$$A = A\cos(k\,x_c + \omega\,t)$$
$$1 = \cos(k\,x_c + \omega\,t)$$

The cosine function is 1 whenever the argument is an integer number of 2π (e.g. $2\pi, 4\pi, 6\pi$, etc.). Write that as n*2π, where "n" is an integer.

$$\cos(k\,x_c + \omega\,t) = 1$$
$$(k\,x_c + \omega\,t) = n * 2\,\pi$$
$$t = \frac{1}{\omega}(n * 2\,\pi - k\,x_c)$$

We don't know which 'n' to pick however. We do know that t must be greater than the time in part (c), which was 0.15 s (because the problem asked for "the next time"). So we can just keep trying 'n' values until the answer is greater than 0.15 s.

```
{ω, k, xc}(* Reminder of these specific numerical values *)
```
Out[44]=
```
{157.08, 19.635, 0.36}
```

In[45]:=
$$\frac{1}{\omega}(1 * 2\,\pi - k\,xc)\ \text{(* n = 1 *)}$$
Out[45]=
```
-0.005
```

In[46]:= $\frac{1}{\omega}(2 * 2\pi - k\,xc)$ (* n = 2 *)

Out[46]= 0.035

In[47]:= $\frac{1}{\omega}(3 * 2\pi - k\,xc)$ (* n = 3 *)

Out[47]= 0.075

In[48]:= $\frac{1}{\omega}(4 * 2\pi - k\,xc)$ (* n = 4 *)

Out[48]= 0.115

In[49]:= $\frac{1}{\omega}(5 * 2\pi - k\,xc)$ (* n = 5 *)

Out[49]= 0.155

The time 0.155 seconds is the first value after part (c). It corresponds to the integer ‘n’ = 5.