Physics 112: General Physics II: Electricity, Magnetism, and Optics
Electron Energy Levels in a One-Dimensional Box

28.71 Two adjacent allowed energies of an electron in a one-dimensional box are 2.00 eV and 4.50 eV. What is the length of the box? *Hint:* Consider ratios of the energies and use Eq. 28.15.

Bonus question: If the electron makes a transition between these two states, what is the wavelength of the resulting photon?

28.71 Two adjacent allowed energies of an electron in a one-dimensional box are 2.00 eV and 4.50 eV. What is the length of the box? *Hint:* Consider the square root of the ratios of the energies and use Eq. 28.15.

We are given two energy states, call them a, and b, where $E_a = 2.00\,\text{eV}$ and $E_b = 4.50\,\text{eV}$. First, recall how energies are quantized in a one-dimensional box. The length of the box L must be an integer number of half-wavelengths. Look at state a:

$$L = n_a \frac{\lambda_a}{2} \implies \lambda_a = \frac{2L}{n_a}$$

$$p_a = \frac{h}{\lambda_a} = \frac{h}{\frac{2L}{n_a}} = \frac{hn_a}{2L}$$

$$E_a = \frac{p_a^2}{2m} = \frac{\left(\frac{hn_a}{2L}\right)^2}{2m} = \left(\frac{h^2}{8mL^2}\right) n_a^2$$

$$E_a = n_a^2 E_1, \text{ where } E_1 = \frac{h^2}{8mL^2}$$

(We have re-derived Eqs. 28.14 and 28.15.) Similarly, the "adjacent" energy E_b is

$$E_b = n_b^2 E_1 = (n_a + 1)^2 E_1$$

where we have used the fact that the states are adjacent to replace n_b by $n_a + 1$.

There are two unknowns: L and n_a. Use the hint:

$$\frac{E_b}{E_a} = \frac{(n_a+1)^2 E_1}{n_a^2 E1} = \frac{(n_a+1)^2}{n_a^2}$$

$$\sqrt{\frac{E_b}{E_a}} = \frac{n_a+1}{n_a}$$

$$\sqrt{\frac{4.50\,\text{eV}}{2.00\,\text{eV}}} = \frac{n_a+1}{n_a}$$

$$1.50 = \frac{n_a+1}{n_a}$$

$$1.50 n_a = n_a + 1 \implies n_a = \frac{1}{1.50 - 1} = 2$$

Thus $n_a = 2$ and $n_b = 3$. Solving for L, and converting the energy E_a to Joules:

$$E_a = (2.00\,\text{eV}) \times (1.60 \times 10^{-19}\,\text{J/eV}) = 3.20 \times 10^{-19}\,\text{J}$$

$$E_a = \left(\frac{h^2}{8mL^2}\right) n_a^2 \implies L^2 = \frac{h^2 n_a^2}{8mE_a}$$

$$L = \frac{hn_a}{\sqrt{8mE_a}} = \frac{(6.63 \times 10^{-34}\,\text{J s}) \times (2)}{\sqrt{8(9.11 \times 10^{-31}\,\text{kg}) \times (3.20 \times 10^{-19}\,\text{J})}}$$

$$L = \boxed{0.867\,\text{nm}}$$

Bonus question: If the electron makes a transition between these two states, what is the wavelength of the resulting photon?

$$\Delta E = \frac{hc}{\lambda} \implies \lambda = \frac{hc}{\Delta E}$$
$$\lambda = \frac{1240\,\text{eV nm}}{2.50\,\text{eV}} = \boxed{496\,\text{nm}}$$