56. ||| A sample of wood from an archaeological excavation is
BIO dated by using a mass spectrometer to measure the fraction of ^{14}C atoms. Suppose 100 atoms of ^{14}C are found for every 1.0×10^{15} atoms of ^{12}C in the sample. What is the wood's age?

Also

Given: Normally, $\frac{N_{^{14}C}}{N_{^{12}C}} = 1.3 \times 10^{-12}$

for ^{14}C, $t_{1/2} = 5730$ yrs, $\tau = \frac{t_{1/2}}{0.693} = 8270$ yrs

Also

Given: Normally, $\frac{N_{^{14}C}}{N_{^{12}C}} = 1.3 \times 10^{-12}$

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This problem considers ratios

Let N_{14} = # of ^{14}C atoms

$N_{14}(0)$ = original # of ^{14}C atoms

Then

$$N_{14} = N_{14}(0)\, e^{-t/\tau}$$

What about ^{12}C? That is stable, so

$N_{12} = N_{12}(0)$ = original # of ^{12}C atoms.

Normally, in living matter,

$$\frac{N_{14}(0)}{N_{12}(0)} = 1.3 \times 10^{-12}$$ (assume this is given.)

After death, the supply of ^{14}C is no longer refreshed, so N_{14} decreases, while N_{12} stays the same. Consider the ratio:

$$\frac{N_{14} = N_{14}(0)\, e^{-t/\tau}}{N_{12} = N_{12}(0)}$$

$$\frac{N_{14}}{N_{12}} = \frac{N_{14}(0)}{N_{12}(0)} e^{-t/\tau}$$

$\frac{N_{14}}{N_{12}}$: This is given in the problem as $\frac{100}{10^{15}} = \frac{1}{10^{13}}$

$\frac{N_{14}(0)}{N_{12}(0)}$: This is given in the problem as $\frac{1.3}{10^{12}}$

$$\therefore \quad \frac{1}{10^{13}} = \left(1.3 \times 10^{-12}\right) e^{-t/\tau}$$

$$0.0769 = e^{-t/\tau}$$

$$\ln(0.0769) = -t/\tau$$

$$t = -\tau \ln(0.0769)$$

$$t = (8270 \text{ yrs})(2.565) = \boxed{21,200 \text{ yrs}}$$

30.61, 3rd edition.

$\boxed{30.56}$ $\dfrac{N_{14}}{N_{12}} = \dfrac{100}{10^{15}} = 10^{-13}$ Don't memorize this!

of ^{14}C

of ^{12}C

$\dfrac{N(0)_{C14}}{N(0)_{C12}} = 1.3\times10^{-12}$ normally. at a later time $N_{12} = N(0)_{12}$, unchanged.

$N_{14} = N_{14}(0)\, e^{-t/\tau}$ $\quad t_{1/2} = \tau \ln 2$

$N_{C12} = N_{C12}(0)$, $\quad \tau = \frac{1}{\ln 2} t_{1/2} = 8266.6$ yrs

$t_{1/2} = 5730$ years

$\tau = 8266.6$ yrs.

$\dfrac{N_{14}}{N_{12}} = 10^{-13} = 1.3\times10^{-12}\, e^{-t/\tau}$

$0.0769 = e^{-t/\tau}$

$\ln(0.0769) = -t/\tau$

$(8266.6\text{yrs})(-2.5649) = -t$

$21,200 \text{ yrs.} = t$