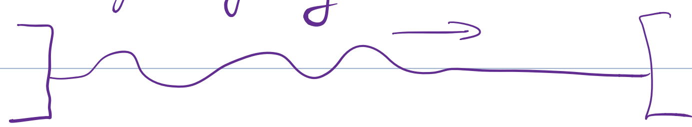
Two ideas:

1) Physics sets wave speed v
2) Boundary conditions set allowed λ

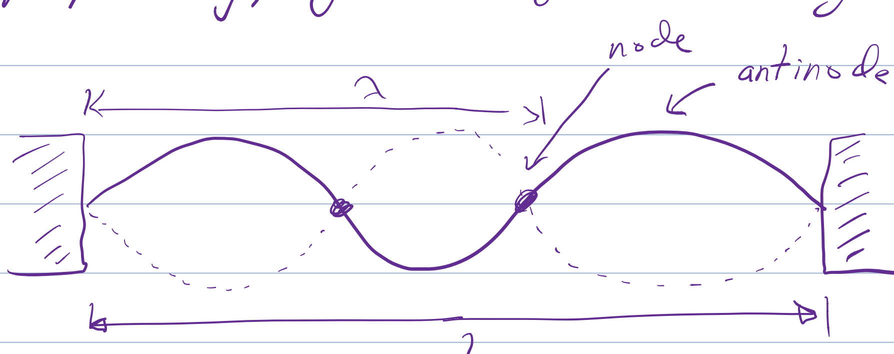
Relate them with $v = \lambda f$.

Make a right-going wave



It reflects back

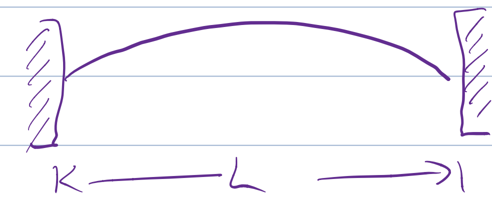
If the f and v values are chosen appropriately, you can get a standing wave



Particles move up and down in simple harmonic motion.

Not all f's work — need nodes at each end. Use pictures.

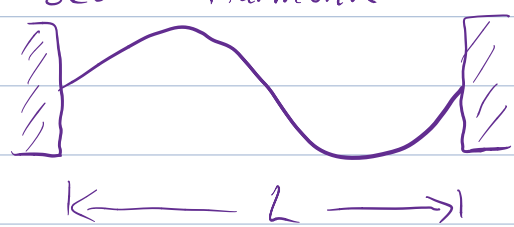
Fundamental



$L = \frac{1}{2}\lambda_1 \Rightarrow \lambda_1 = 2L$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

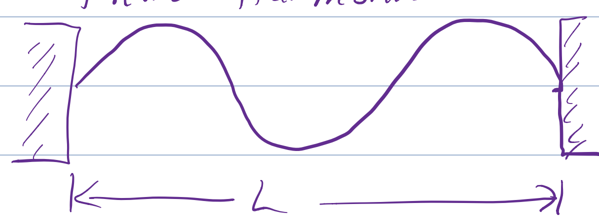
Second Harmonic



$L = \frac{2}{2}\lambda_2 \Rightarrow \lambda_2 = \frac{2L}{2}$

$$f_2 = \frac{v}{\lambda_2} = 2\frac{v}{2L}$$

Third Harmonic



$L = \frac{3}{2}\lambda_3 \Rightarrow \lambda_3 = \frac{2L}{3}$

$$f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L}$$

General idea:

$$\boxed{\begin{aligned} L &= (integer)\cdot\frac{\lambda_n}{2} = \frac{n\lambda_n}{2} \\ f_n &= \frac{v}{\lambda_n} = n\frac{v}{2L}; \end{aligned}}$$

Use pictures to find λ.

Use $v = \sqrt{F_T/\mu}$ to find v.

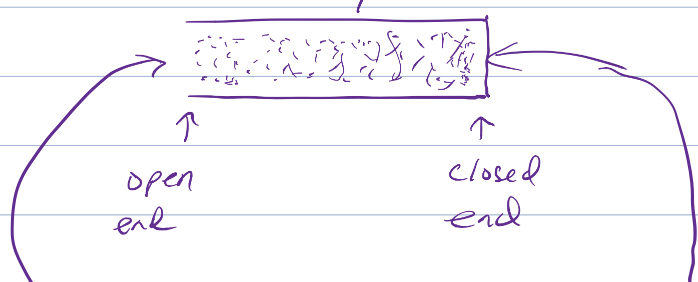
Relate v, f, λ by $v = \lambda f$.

See examples. Actual motion can be a superposition of these modes of oscillation.

16.4 Standing Sound Waves

Strings were clamped at both ends.
Sound tubes can be open or closed.

Think about pressure variations



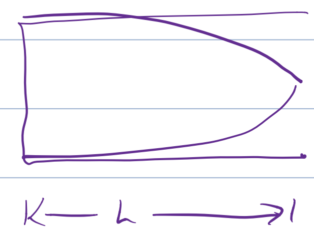
p here is always 1 atmosphere.

p here can vary. This will be a pressure antinode.

open end = pressure node.

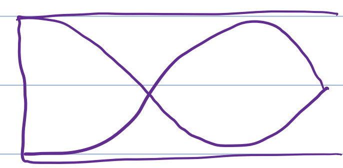
Standing waves: Only certain λ's will satisfy the boundary conditions.
Draw pictures to get λ.

open — closed



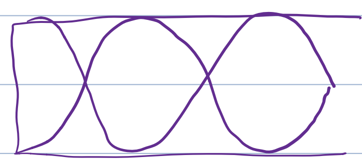
$$L = \frac{1}{4}\lambda_1 \Rightarrow \lambda_1 = \frac{4L}{1}$$

$$f_1 = v/\lambda_1 \Rightarrow f_1 = 1\,\frac{v}{4L}$$



$$L = \frac{3}{4}\lambda_3 \Rightarrow \lambda_3 = \frac{4L}{3}$$

$$f_3 = v/\lambda_3 = \frac{3v}{4L}$$



$$L = \frac{5}{4}\lambda_5 \Rightarrow \lambda_5 = \frac{4L}{5}$$

$$f_5 = v/\lambda_5 = 5\,\frac{v}{4L}$$

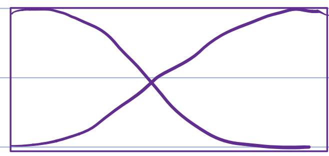
Allowed frequencies: $f_1, f_3, f_5 \ \ldots$

$$f_n = (\text{odd})\,\frac{v}{4L}$$

Don't memorize!

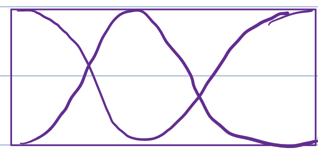
Draw pictures!

closed – closed (pressure antinodes at each end)



$L = \frac{1}{2}\lambda_1 \Rightarrow \lambda_1 = \frac{2L}{1}$

$f_1 = v/\lambda_1 = 1\,\frac{v}{2L}$



$L = \frac{2}{2}\lambda_2 \Rightarrow \lambda_2 = \frac{2L}{2}$

$f_2 = v/\lambda_2 = 2\,\frac{v}{2L}$

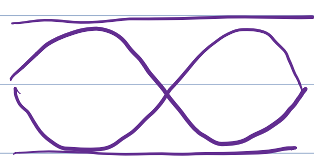
Allowed frequencies: $f_1, f_2, f_3, \ldots$

$$f_n = n\,\frac{v}{2L}.$$

open – open (pressure nodes at each end)

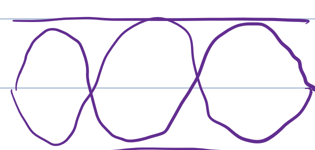
$L = \frac{1}{2}\lambda_1 \Rightarrow \lambda_1 = \frac{2L}{1}$

$f_1 = v/\lambda_1 = 1\,\frac{v}{2L}$



$L = \frac{2}{2}\lambda_2 \Rightarrow \lambda_2 = \frac{2L}{2}$

$f_2 = v/\lambda_2 = 2\,\frac{v}{2L}$



$L = \frac{3}{2}\lambda_3 \Rightarrow \lambda_3 = \frac{2L}{3}$

$f_3 = \frac{v}{\lambda_3} = 3\,\frac{v}{2L}$

$$f_n = n\frac{v}{2L}$$

A potentially confusing complication: pressure vs displacement for sound waves

At an open end:

pressure = constant $\Rightarrow$ node

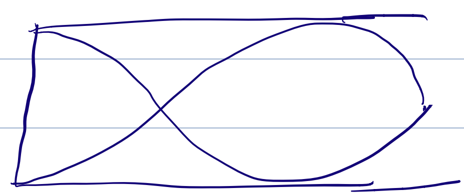
displacement $\rightarrow$ varies a lot $\Rightarrow$ antinode

At a closed end:

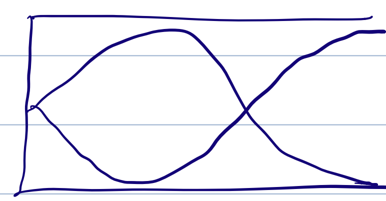
pressure $\rightarrow$ varies $\Rightarrow$ antinode

displacement = 0 $\Rightarrow$ node.

pressure picture
(our text)



displacement picture
(previous text)



Key: it doesn't matter which you pick! $L = (\text{odd})\dfrac{\lambda}{4}$ still works.

16.5 Speech and Hearing. OMIT