Superposition of Traveling Waves
Physics 112

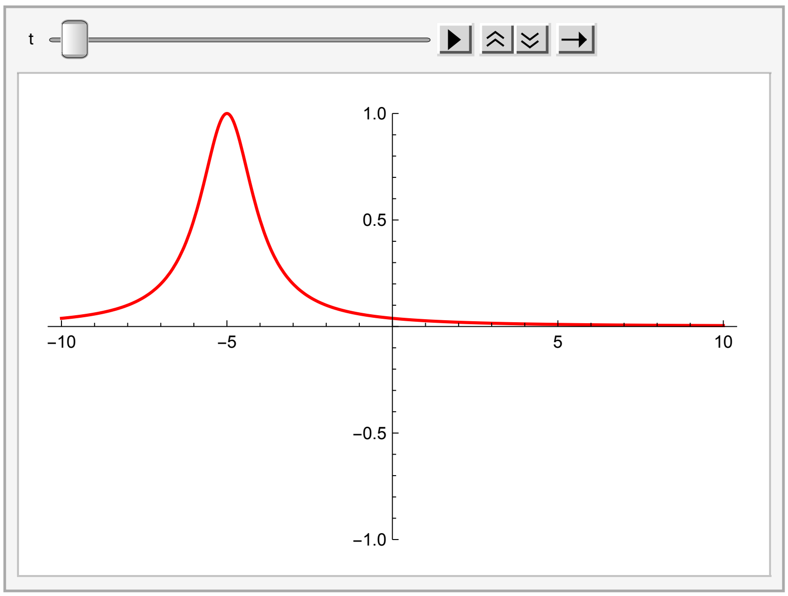
In[42]:=
```
Clear["Global`*"]
```

Two wave pulses traveling in opposite directions

In[43]:=
```
p[x_, x0_] := 1 / (1 + (x - x0) ^2) (* A pulse centered at x0 *)
v = 2;
p1[x_, t_] :=  p[x - v t, -5]
p2[x_, t_] := -p[x + v t, 5]
```

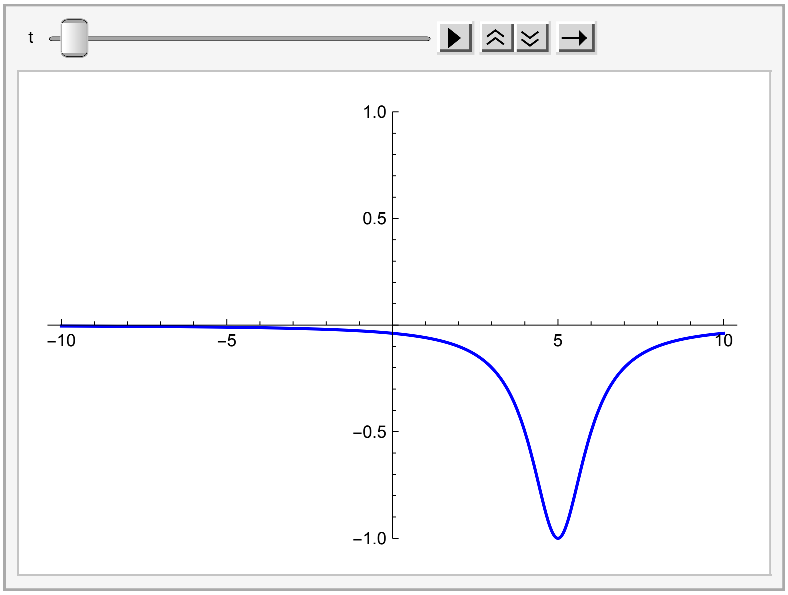
In[47]:=
```
Animate[Plot[p1[x, t], {x, -10, 10}, PlotStyle → Red, PlotRange → {-1, 1}],
 {t, 0, 5}, AnimationRunning → False, DefaultDuration → 5]
```

Out[47]=



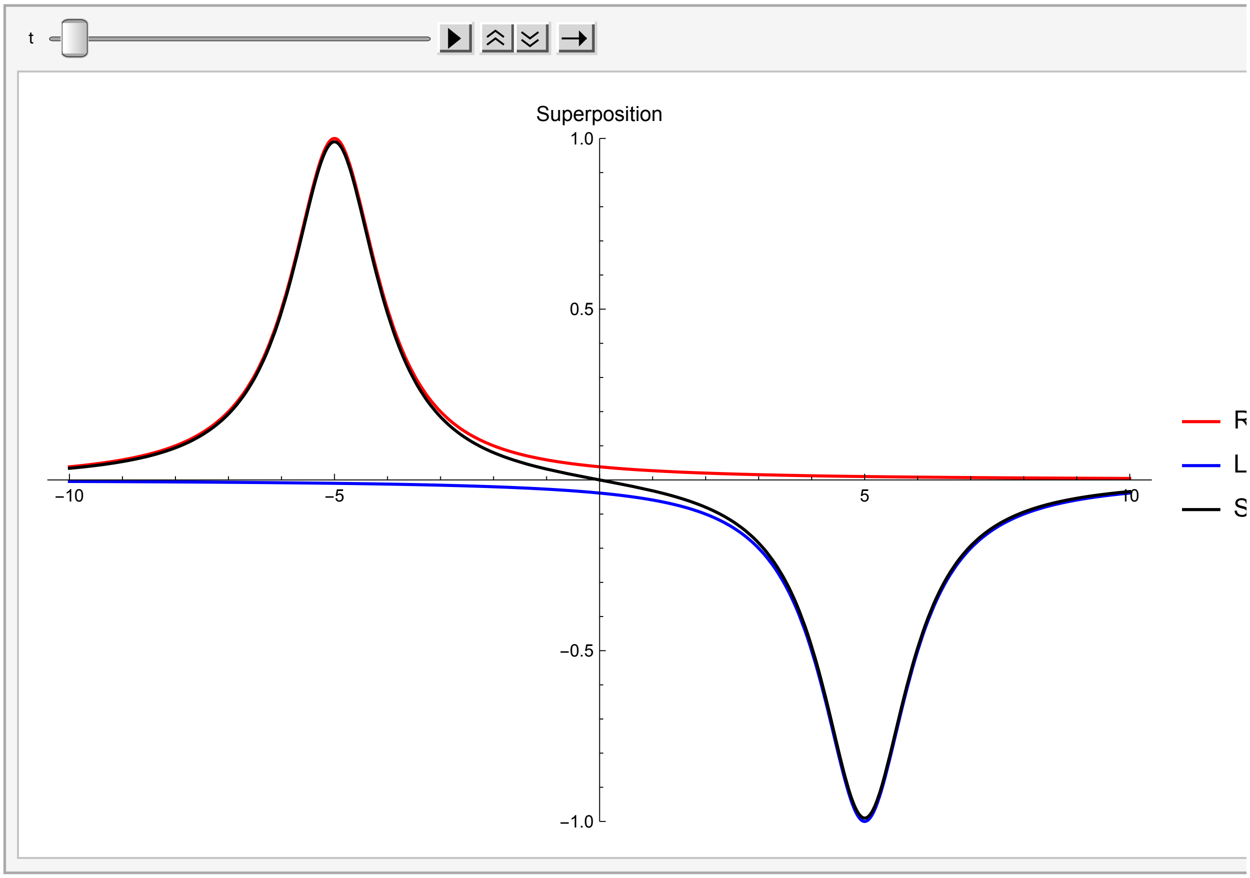
In[48]:=
```
Animate[Plot[p2[x, t], {x, -10, 10}, PlotStyle → Blue, PlotRange → {-1, 1}],
 {t, 0, 5}, AnimationRunning → False, DefaultDuration → 5]
```

Out[48]=



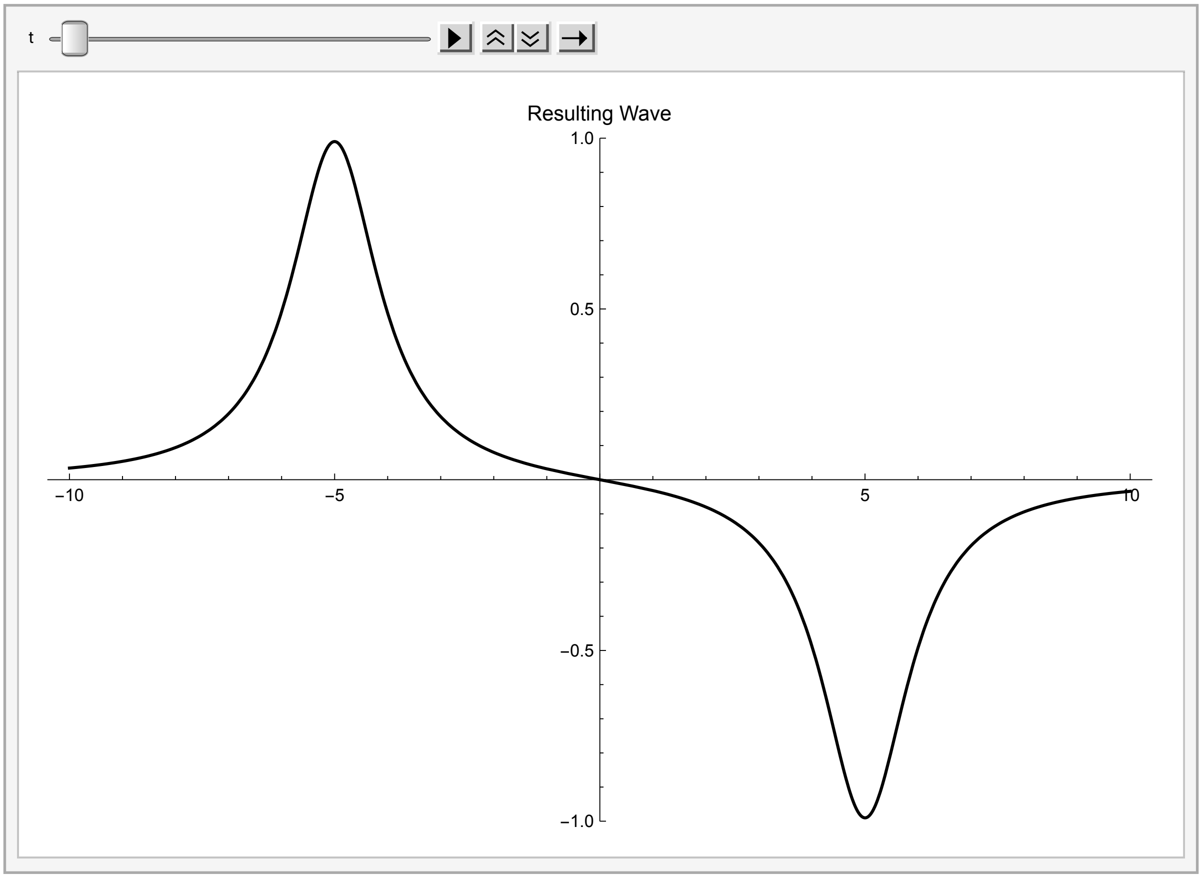
In[49]:=
```
Animate[Plot[{
    p1[x, t], p2[x, t], p1[x, t] + p2[x, t]}, {x, -10, 10},
  PlotStyle → {Red, Blue, Black}, PlotLegends → {"Right", "Left", "Sum"},
  PlotRange → {-1, 1}, ImageSize → Large, PlotLabel → "Superposition"],
 {t, 0, 5}, AnimationRunning → False, DefaultDuration -> 15]
```

Out[49]=



In[50]:=
```
Animate[
 Plot[p1[x, t] + p2[x, t], {x, -10, 10}, PlotStyle → Black,
  PlotRange → {-1.0, 1.0}, ImageSize → Large, PlotLabel → "Resulting Wave"],
 {t, 0, 5}, AnimationRunning → False, DefaultDuration → 10]
```

Out[50]=



Two sine waves traveling to the right

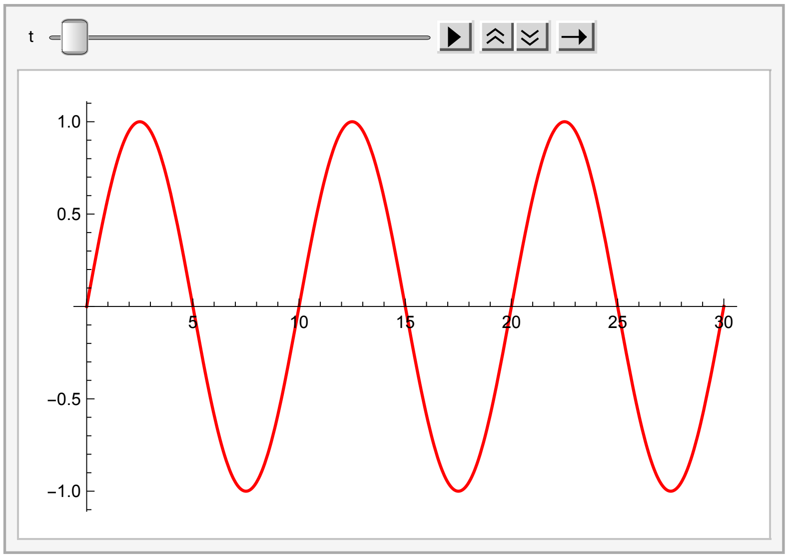
Consider some simple waves, each of amplitude 1, wavelength λ, and wave speed v.

In[51]:=
```
y1[λ_, f_, x_, t_] := Sin[(2 π / λ) x - (2 π f) t]
y2[λ_, f_, x_, t_] := Sin[(2 π / λ) x - (2 π f) t + π / 2]
 (* Add a phase to make it interesting *)
```

In[53]:=
```
λ = 10;
f = 4;
T = 1 / f;
L = 8 * (λ / 2);
```

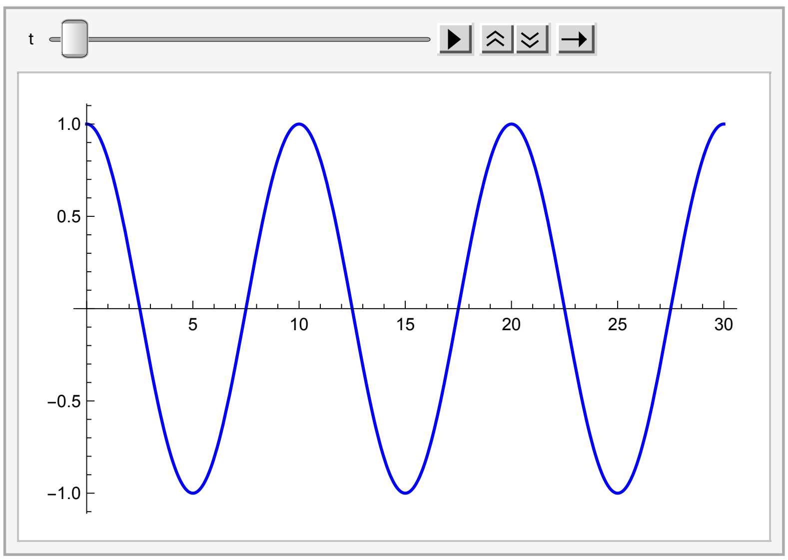
In[57]:=
```
Animate[Plot[y1[λ, f, x, t], {x, 0, L}, PlotStyle → Red],
 {t, 0, 5 * T}, AnimationRunning → False, DefaultDuration → 5]
```

Out[57]=



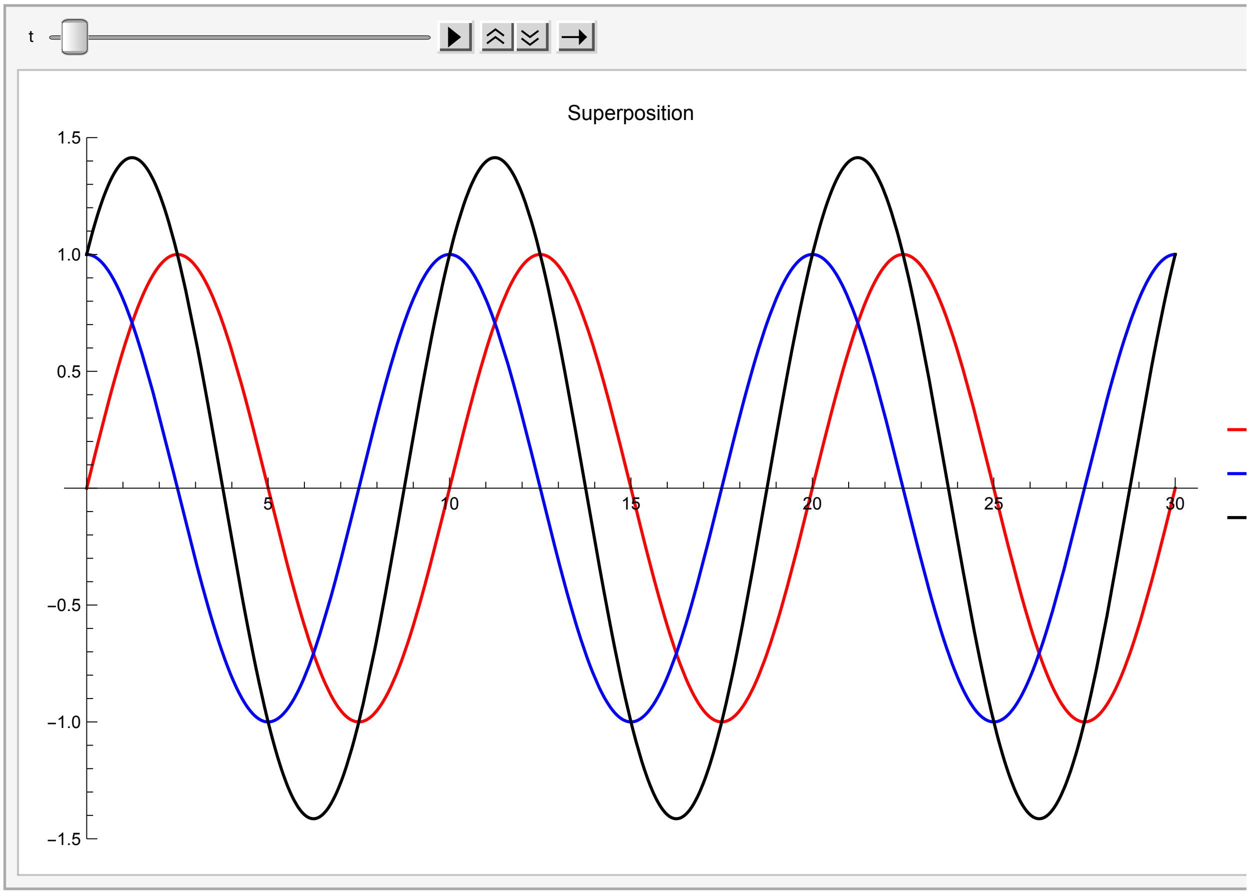
In[58]:=
```
Animate[Plot[y2[λ, f, x, t], {x, 0, L}, PlotStyle → Blue],
 {t, 0, 5 * T}, AnimationRunning → False, DefaultDuration → 5]
```

Out[58]=



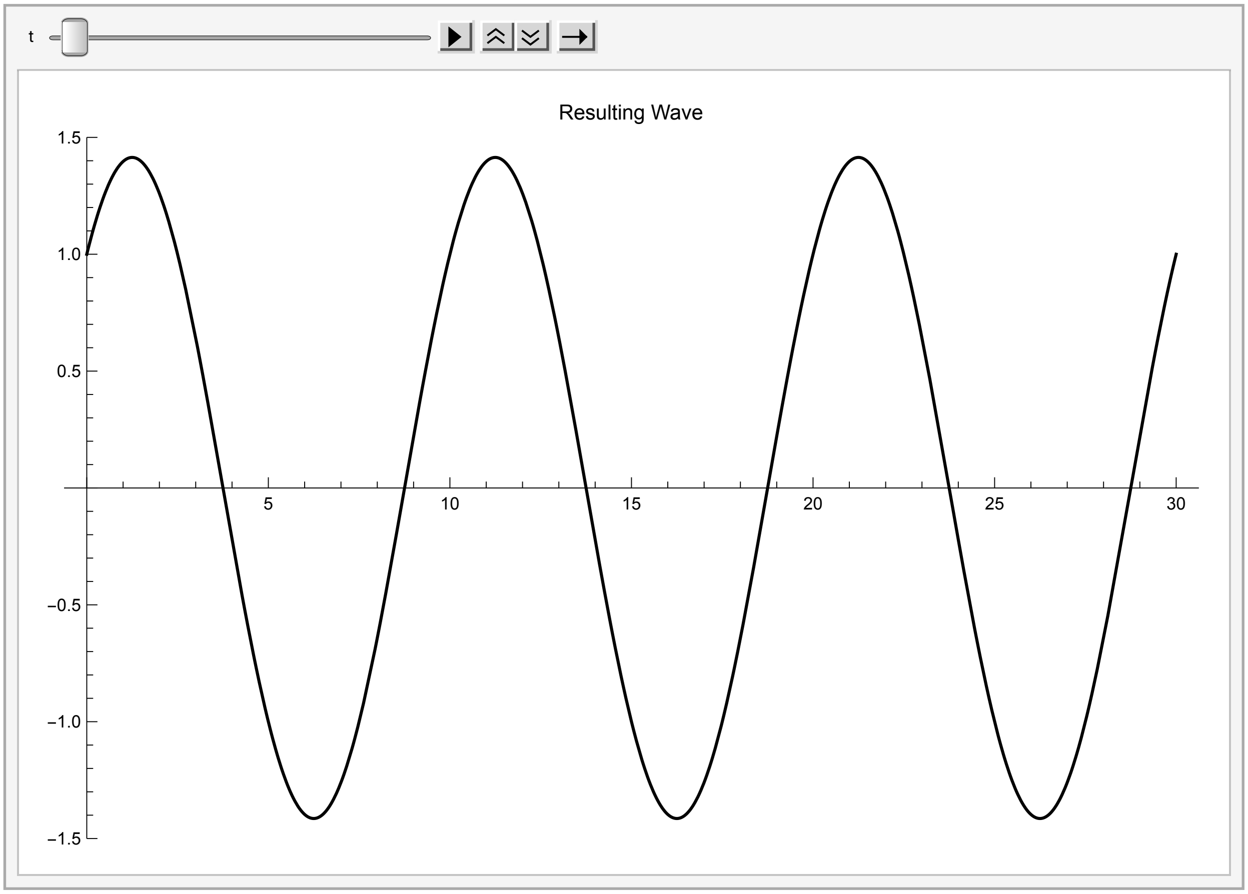
In[59]:=
```
Animate[
 Plot[{y1[λ, f, x, t], y2[λ, f, x, t], y1[λ, f, x, t] + y2[λ, f, x, t]},
  {x, 0, L}, PlotStyle → {Red, Blue, Black}, PlotLegends → {"1", "2", "Sum"},
  PlotRange → {-1.5, 1.5}, ImageSize → 600, PlotLabel → "Superposition"],
 {t, 0, 5 * T}, AnimationRunning → False, DefaultDuration -> 15]
```

Out[59]=



In[60]:=
```
Animate[
 Plot[y1[λ, f, x, t] + y2[λ, f, x, t], {x, 0, L}, PlotStyle → Black,
  PlotRange → {-1.5, 1.5}, ImageSize → 600, PlotLabel → "Resulting Wave"],
 {t, 0, 5 * T}, AnimationRunning → False, DefaultDuration → 10]
```

Out[60]=



Left and Right-going waves

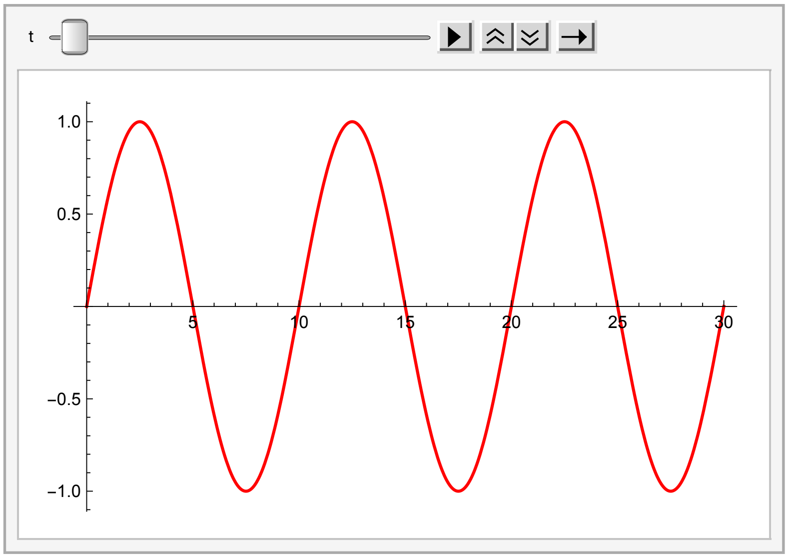
Assume the wave is confined to a length L. To get standing waves, we will see that we need to have L = (integer) * (λ/2)

In[61]:=
```
yr[λ_, f_, x_, t_] := Sin[(2 π / λ) x - (2 π f) t]
yl[λ_, f_, x_, t_] := Sin[(2 π / λ) x + (2 π f) t]
```

In[63]:=
```
λ = 10;
f = 4;
T = 1 / f;
L = 6 * (λ / 2);
```

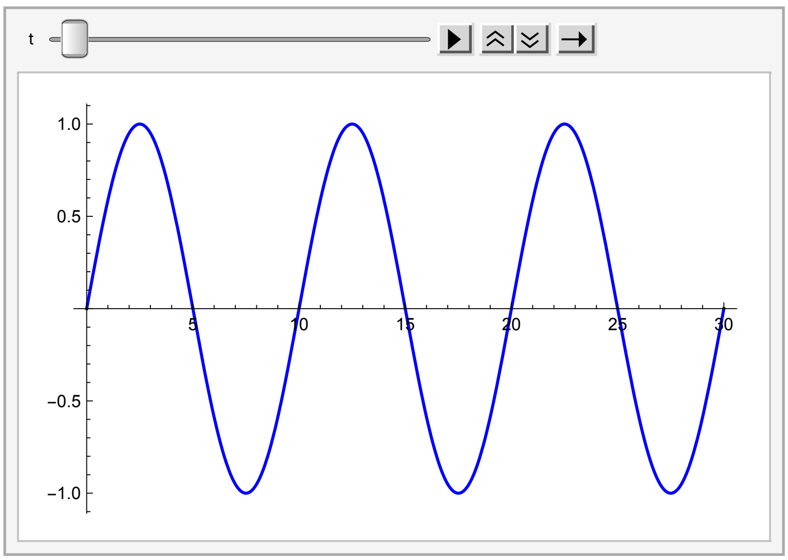
In[67]:=
```
Animate[Plot[yr[λ, f, x, t], {x, 0, L}, PlotStyle → Red],
 {t, 0, 5 * T}, AnimationRunning → False, DefaultDuration → 5]
```

Out[67]=



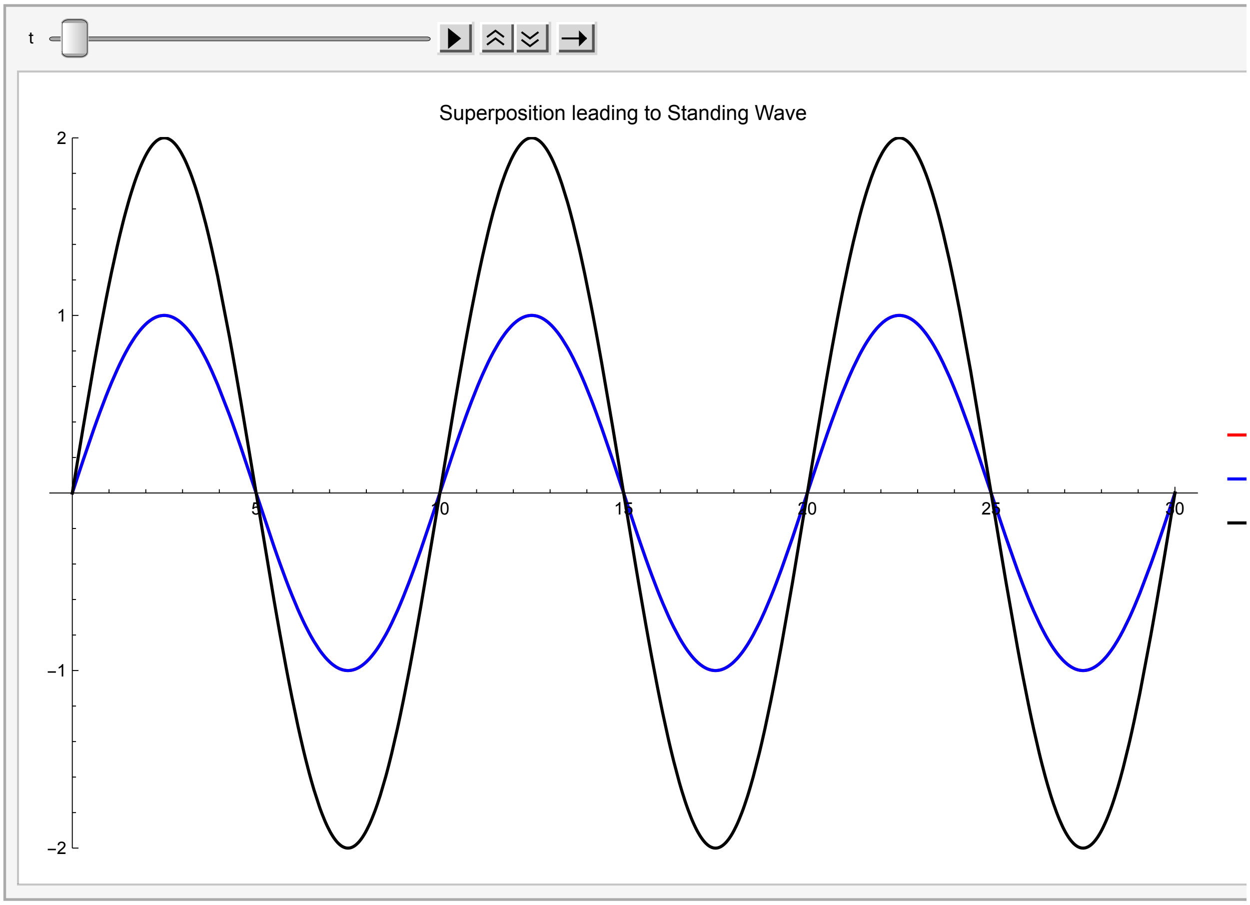
In[68]:=
```
Animate[Plot[yl[λ, f, x, t], {x, 0, L}, PlotStyle → Blue],
 {t, 0, 5 * T}, AnimationRunning → False, DefaultDuration → 5]
```

Out[68]=



In[69]:=
```
Animate[Plot[{yr[λ, f, x, t], yl[λ, f, x, t], yr[λ, f, x, t] + yl[λ, f, x, t]},
   {x, 0, L}, PlotLegends → {"Right", "Left", "Sum"},
   PlotStyle → {Red, Blue, Black}, PlotRange → {-2, 2}, ImageSize → 600,
   PlotLabel → "Superposition leading to Standing Wave"],
  {t, 0, 5 * T}, AnimationRunning → False, DefaultDuration -> 15]
```

Out[69]=



In[70]:=
```
Animate[
 Plot[yr[λ, f, x, t] + yl[λ, f, x, t], {x, 0, L}, PlotStyle → Black,
  PlotRange → {-2, 2}, ImageSize → 600, PlotLabel → "Standing Wave"],
 {t, 0, 5 * T}, AnimationRunning → False, DefaultDuration → 10]
```

Out[70]=