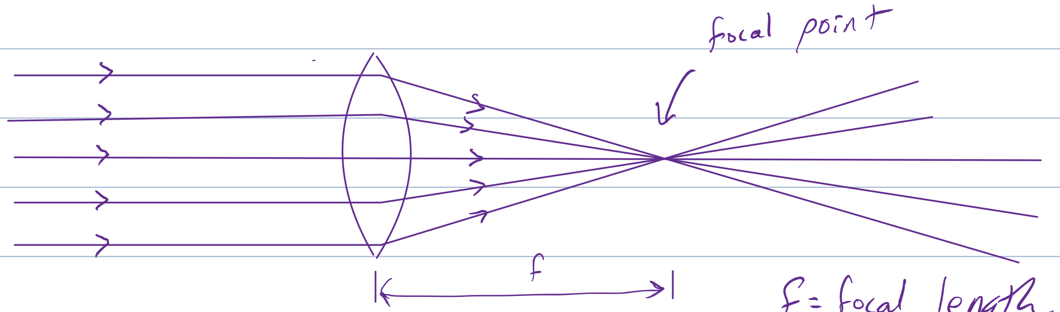
18.5 Thin Lenses: Ray Tracing

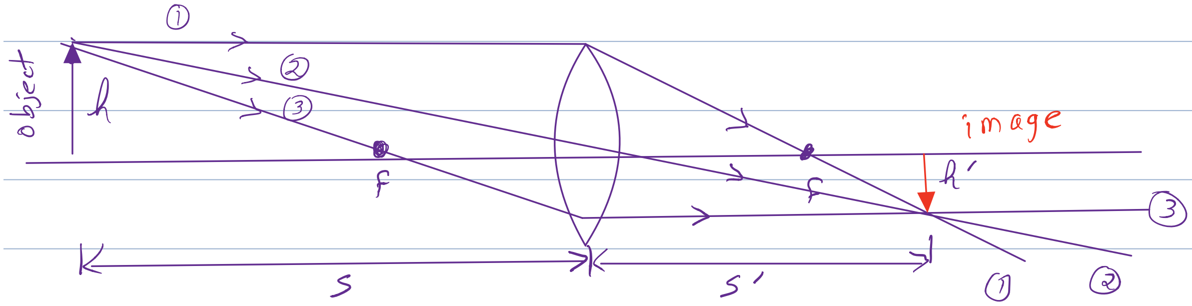
Look at two different types of lenses

1) Converging : parallel rays converge at a focal point



f = focal length

Image formation



Trace 3 rays

① ray parallel to the axis goes out through the focal point.

② ray through the center is undeflected

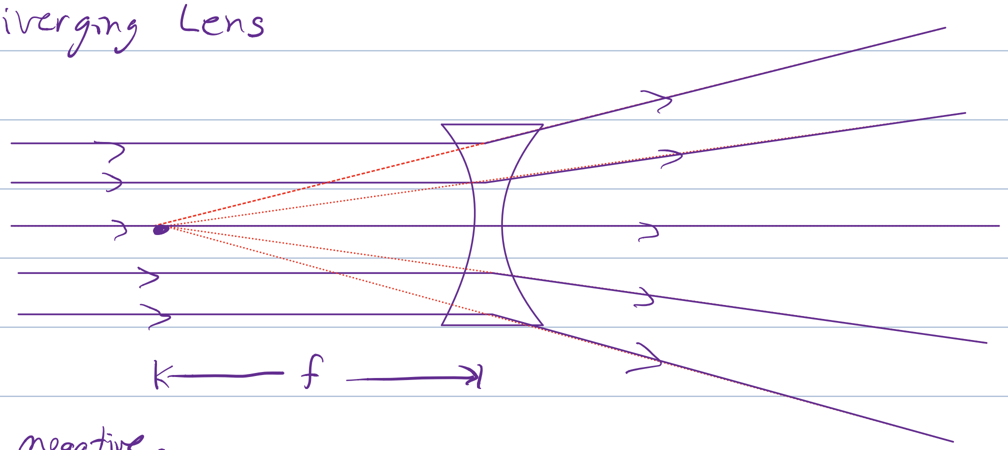
③ ray going in through the focal point comes out parallel to the axis.

s = object distance   s' = image distance

h = object height   h' = image height

(more details in next section)

2) Diverging Lens



f is negative.

18.7: The Thin Lens Equation

How are these all related ?

Thin lens equation:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$ gives location

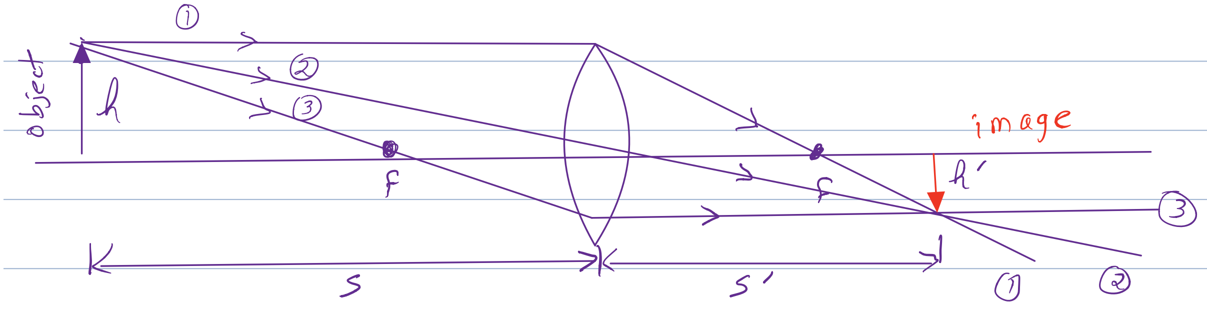
magnification $m = \frac{h'}{h} = \frac{-s'}{s}$ (- sign ⇒ inverted)

For our converging lens configuration above, f, s, h, and s' are all positive.

h' is negative, since the image is inverted.

Next — walk through some standard configurations.

1. Canonical case:



e.g. $f = 10\text{ cm}$ $s = 24\text{ cm}$ $h = 3\text{ cm}$.

where is the image?

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{1}{10\text{cm}}-\frac{1}{24\text{cm}}$$

$s' = 17.1\text{ cm}$

How tall is it?

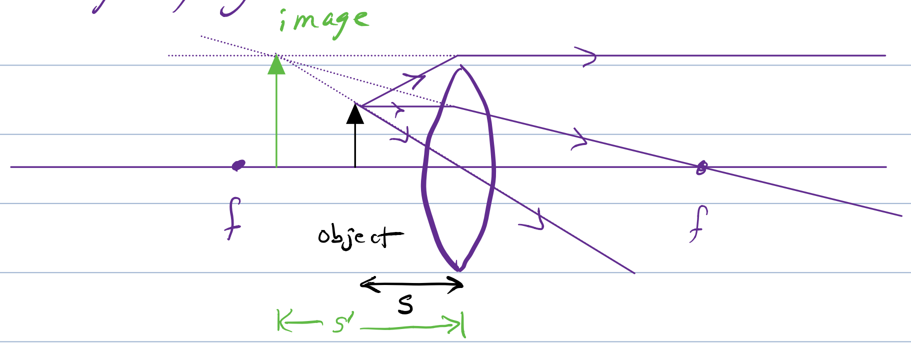
$$m=\frac{h'}{h}=\frac{-s'}{s} \Rightarrow h'=-h\left(\frac{s'}{s}\right)$$

$$h'=-(3\text{ cm})\left(\frac{17.1\text{cm}}{24\text{cm}}\right)=\underset{\substack{\uparrow\\ \text{inverted}}}{-}\underbrace{2.14\text{ cm}}_{\text{smaller}}$$

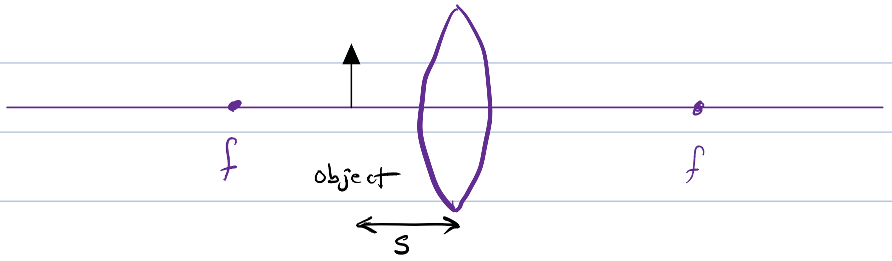
OR... $m=\frac{-s'}{s}=\frac{-17.1\text{ cm}}{24\text{ cm}}=\underset{\substack{\uparrow\\ \text{inverted.}}}{-}\underbrace{0.714}_{\text{smaller}}$

We say there is a <u>REAL</u> image → the rays really converge at s'. If you put a screen there, you will see the image.

2. Magnifying Glass



Redraw:



example: $f = 10\text{ cm}$ $s = 4\text{ cm}$ $h = 1\text{ cm}$

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \Rightarrow \frac{1}{s'} = \frac{1}{f} - \frac{1}{s}$$

$$\frac{1}{s'} = \frac{1}{10\text{cm}} - \frac{1}{4\text{ cm}} = \frac{-3}{20\text{cm}} \Rightarrow s' = \frac{-20}{3}\text{cm} \approx -6.7\text{cm}$$

what does this mean?

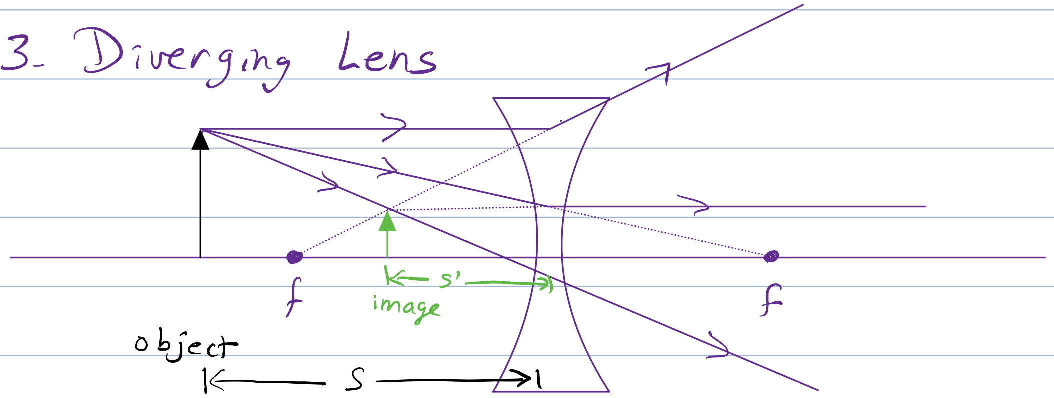
- sign ⇒ on "wrong" side of the lens,
  i.e. same side as object.

$$m = \frac{h'}{h} = \frac{-s'}{s} = \frac{-(-6.7\text{ cm})}{4\text{ cm}} = +1.67$$

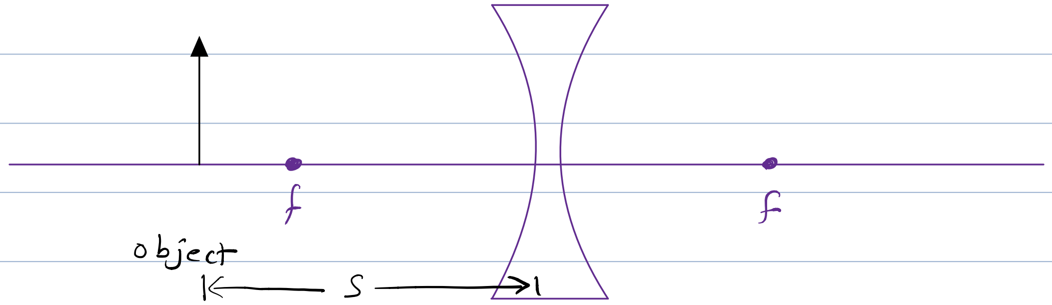
$\therefore h' = mh = 1.67\text{ cm}$. Positive ⇒ upright

This is a <u>virtual</u> image. No light rays are actually converging at s'.

3. Diverging Lens



(copy)



example: $f = -10$ cm (diverging) $s = 14$ cm
$h = 3$ cm

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \Rightarrow \frac{1}{s'} = \frac{1}{f} - \frac{1}{s}$$

$$\frac{1}{s'} = \frac{1}{-10\text{ cm}} - \frac{1}{14\text{ cm}} = \frac{-12}{70\text{ cm}} \Rightarrow s' = -5.8\text{ cm}$$

$-$ sign ⇒ same side of lens as object.

$$m = \frac{h'}{h} = -\frac{s'}{s} = \frac{-(-5.8)}{14} = +0.42$$

$h' = (0.42)(3\text{ cm}) = 1.25$ cm.

i.e. smaller but closer. Virtual image.

This is how eyeglasses for nearsighted people work — the image is closer than the actual object.

Alternate way to categorize a lens

Power in Diopters $P \equiv \frac{1}{f \text{ (in meters)}}$

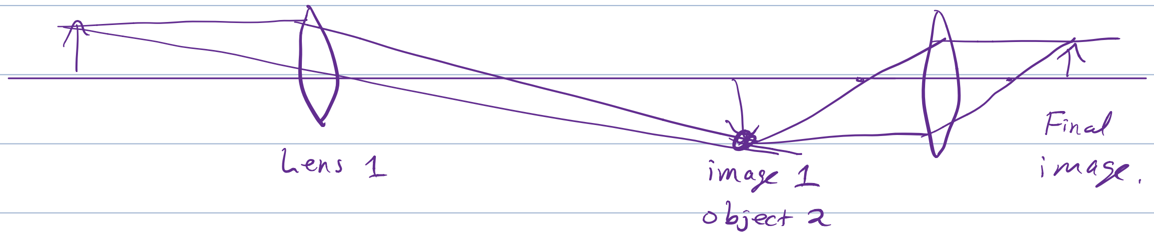
e.g. -1.25 diopters $\Rightarrow f = \frac{1}{-1.25} = -0.8\,m$

-6 diopters $\Rightarrow f = \frac{1}{-6} = -0.167\,m$

$+2$ diopters $\Rightarrow f = \frac{1}{2} = +0.5\,m$

Lens Combinations:

image from first lens serves as object for the second lens.



Ray tracing is hard.

See examples

Applications — Ch. 19.

I8.6 Mirrors - 0MIT