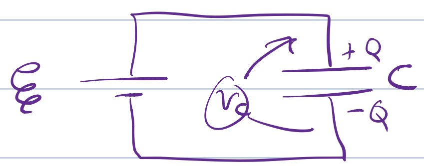
23.6 capacitors in series and parallel

Capacitor circuit symbol: —||— reminds us of the parallel plates.

Basic capacitor circuit:

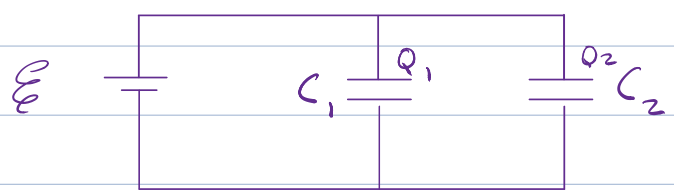


In steady state, no current flows! Capacitor is charged up to $V_c = \mathcal{E}$.

Recall $Q = C\,(\Delta V)$

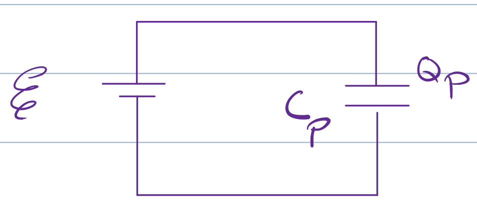
$Q = C\mathcal{E}$

Parallel Combination



$\Longleftrightarrow$

What is the net effective capacitance?



The total charge stored is $Q_p = Q_1 + Q_2$

Use the relation $Q = C(\Delta V)$, while recognizing $\Delta V = \mathcal{E}$ for both capacitors.

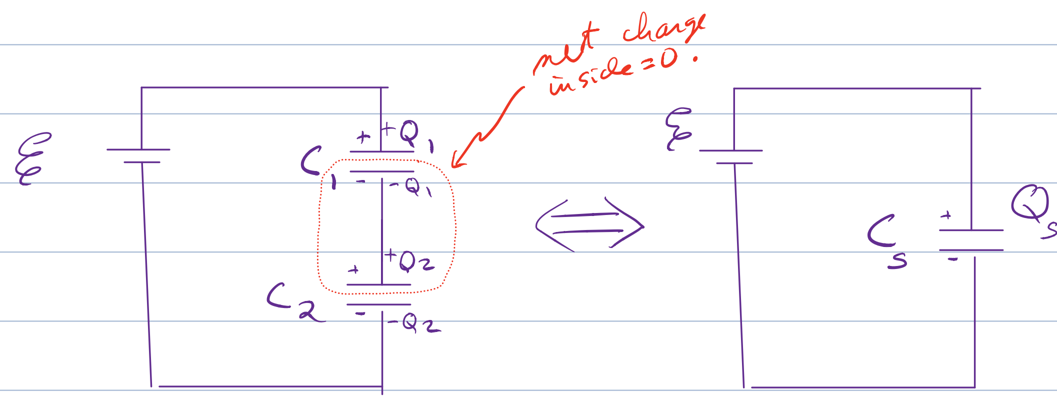
$$Q_p = Q_1 + Q_2$$

$$C_p \mathcal{E} = C_1 \mathcal{E} + C_2 \mathcal{E}$$

$$\therefore \boxed{C_p = C_1 + C_2}$$

recall for parallel plates: $C = \dfrac{\epsilon_0 A}{d}$. we have essentially added more area.

Series combination:



$\therefore Q_1 = Q_2 = Q_s$

KVL: $\mathcal{E} - \Delta V_1 - \Delta V_2 = 0$

$$\frac{Q_s}{C_s} - \frac{Q_1}{C_1} - \frac{Q_2}{C_2} = 0$$

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}}$$

$Q_s = C_s \mathcal{E}$

$\Rightarrow \mathcal{E} = \dfrac{Q_s}{C_s}$

series equivalent. $C = \dfrac{\epsilon_0 A}{d}$, as if we combined distances.

23.7 RC Circuits

Important application: timing.

Units note

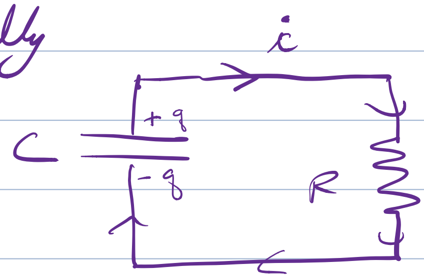
$$[RC] = \Omega \cdot F = \frac{V}{A} \cdot \frac{C}{V} = \frac{1}{C/s} \cdot C = \text{seconds}.$$

Discharge: Consider a capacitor initially charged to voltage V_0, with charge $Q_0 = CV_0$.

close at time $t = 0$.

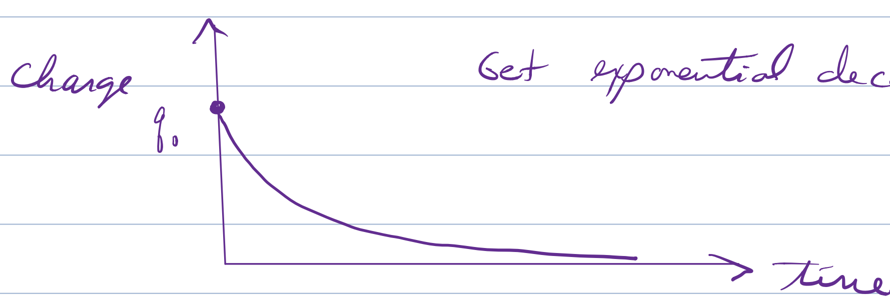
Let q = charge at any instant.

initially



current flows as charges leave + plate.

But ... as charges leave, $\Delta V = q/C$ decreases, so the voltage across the resistor decreases and i decreases.



Get exponential decay

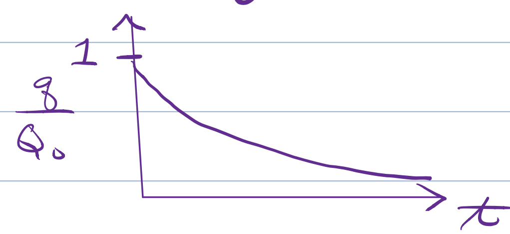
$$q = Q_0 e^{-t/RC}$$

$$q = C V_0 e^{-t/RC}$$

also $$i = \frac{V_0}{R} e^{-t/RC}$$

a bit about exponential decay:

$$\frac{q}{Q_0} = e^{-t/RC}$$



t	q/Q_0	
0	1	
$1 \cdot RC$	$e^{-1} = 1/e = 0.368$	
$2 \cdot RC$	$e^{-2} = 0.135$	
$3 \cdot RC$	$e^{-3} \simeq 0.0498$	$(\sim 5\%)$
$5 \cdot RC$	$e^{-5} \simeq 0.0067$	$(< 1\%)$
⋮	⋮	

RC = "time constant" $= \tau$

e.g. $R = 10\,k\Omega$ $\quad C = 0.01\,nF = 10^{-11}\,F$

$\tau = RC = 10^4\,\Omega \cdot 10^{-11}\,F = 10^{-7}\,s = 0.1\,\mu s$

e.g. $R = 500\,k\Omega$ $\quad C = 5\,\mu F$

$\tau = RC = 500 \times 10^3\,\Omega \cdot 5 \times 10^{-6}\,F = 2.5\,s$

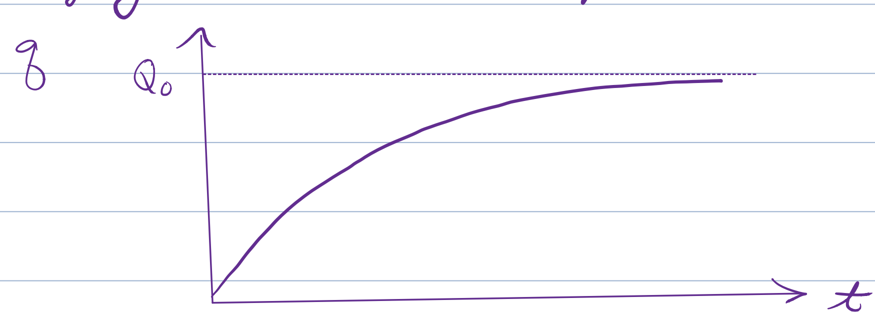
RC circuits are widely used for timing.

Question: how long does it take to discharge?
There is no exact answer. It depends on your precise criteria. after $\sim 3\tau$, 95% is gone. after $\sim 5\tau$, >99% is gone.

Another description: half life. How long does it take to go down to half of what you have (e.g. $6V \to 3V$, or $4V \to 2V$, etc.)

$$q = Q_0 e^{-t/\tau}$$
$$CV = CV_0 e^{-t/\tau}$$
$$\frac{1}{2} V_0 = V_0 e^{-t/\tau}$$
$$\frac{1}{2} = e^{-t/\tau}$$
$$\ln(1/2) = -t/\tau$$
$$\ln 2 = t/\tau$$
$$\boxed{t_{1/2} = \tau \ln 2}$$
, when $\ln 2 \approx 0.693$

Charging: same shape, but upside down.



$$q = Q_0 (1 - e^{-t/RC})$$ Not on test

Application: Store and release energy over particular time interval

Recall $U_c = \frac{1}{2} Q (\Delta V)$

decrease over time $\tau = RC$.