

Ch. 29 Part 2

The Bohr Model of the Hydrogen Atom

29.4 The Bohr Hydrogen Atom

Develop a *simple* solar-system-like model to explore the hydrogen atom. This model is overly simplistic, but still yields some useful insights and helps us visualize other, more complex, atomic systems.

- The nucleus is fixed in place.
- The electron orbits the nucleus in a circular orbit of radius r .
- Only certain radii are observed or “allowed.”
- Those allowed radii correspond to specific allowed energy levels.
- The electron doesn't really follow a strict well-defined orbit, but we can still gain some good insights from this picture.

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Let r_n = average distance of the electron from the nucleus. Quantum mechanics ultimately tells us only certain r 's are observed. Specifically,

$$r_n = n^2 a_0$$

where n = an integer, (1, 2, 3, 4, ...), and a_0 is the Bohr radius, given by

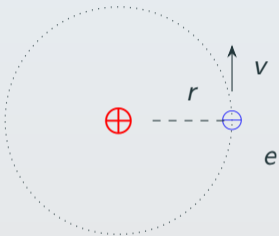
$$a_0 = \frac{h/(2\pi)}{mKe^2} = 0.0529 \text{ nm}$$

where $h = 6.63 \times 10^{-34}$ Js is Planck's constant, $m = 9.11 \times 10^{-31}$ kg is the electron mass, $K = 8.99 \times 10^9$ Nm²/C² is Coulomb's constant, and $e = 1.60 \times 10^{-19}$ C is the electron charge.

Given this one quantum mechanical constraint, the remainder of the calculation is a classical electrostatics calculation. It is a good review of material from Chs. 20 and 21.

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Consider the electron in a circular orbit of radius r .



What is the energy?

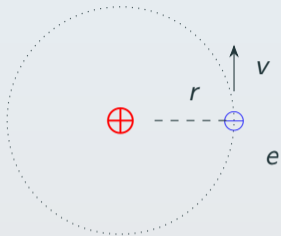
$$E = K + U = \frac{1}{2}mv^2 - \frac{Ke^2}{r}$$

What is the speed? Look at Newton's

second law. The electric attraction between the proton and electron provides the centripetal acceleration:

$$\begin{aligned} F &= ma \\ \frac{Ke^2}{r^2} &= m\frac{v^2}{r} \\ \frac{Ke^2}{r} &= mv^2 \\ \frac{Ke^2}{2r} &= \frac{1}{2}mv^2 \end{aligned}$$

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Recall that the potential energy is $U = -\frac{Ke^2}{r}$, so the final lines above relate the kinetic and potential energies:

$$\frac{Ke^2}{2r} = \frac{1}{2}mv^2$$
$$-\frac{1}{2}U = K$$

Thus the total energy is

$$E = K + U = \left(-\frac{1}{2}U\right) + U = \frac{1}{2}U = -\frac{Ke^2}{2r}$$

E depends on r . Since only certain $r_n = n^2 a_0$ values are observed, only certain energies E_n are observed:

$$E_n = -\frac{Ke^2}{2r_n} = -\frac{Ke^2}{2n^2 a_0}$$
$$E_n = -\frac{1}{n^2} \frac{Ke^2}{2a_0} = -\frac{1}{n^2} E_0$$
$$E_n = -\frac{1}{n^2} (13.6 \text{ eV})$$

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The observed energies of the hydrogen atom are thus

$$E_n = -\frac{1}{n^2} \frac{Ke^2}{2a_0} = -\frac{1}{n^2} E_0 = -\frac{1}{n^2} (13.6 \text{ eV})$$

where the constant $E_0 = 13.6 \text{ eV}$, and n is a positive integer. Transitions between two different levels m and n involve energy jumps

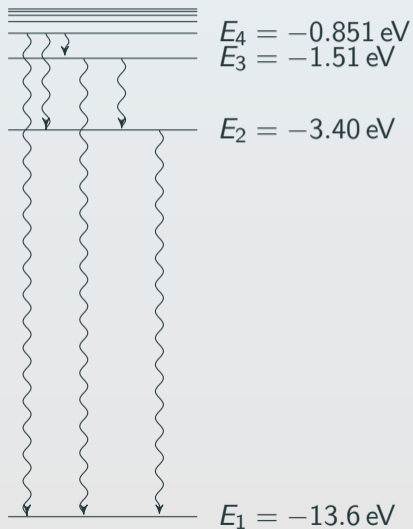
$$\Delta E = E_n - E_m = \left(-\frac{1}{n^2} E_0\right) - \left(-\frac{1}{m^2} E_0\right) = E_0 \left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

This energy difference corresponds to a photon of wavelength λ given by

$$\lambda = \frac{hc}{\Delta E} = \frac{hc}{E_0 \left(\frac{1}{m^2} - \frac{1}{n^2}\right)} = \frac{1240 \text{ eVnm}}{13.6 \text{ eV} \left(\frac{1}{m^2} - \frac{1}{n^2}\right)} = \frac{91.1 \text{ nm}}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)}$$

which is the same experimental result we saw at the beginning of the chapter.

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This formula reproduces the same energy level diagram we had before. The energy levels bunch up as n increases. The ionization limit 0 eV is approached as $n \rightarrow \infty$.

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Some general observations building off

$$E_n = -\frac{1}{n^2}E_0 = -\frac{13.6 \text{ eV}}{n^2}$$

- The total energies are negative, but become less so for larger r .
- To move the electron away from the proton, you have to do work, or add energy
- If $r \rightarrow \infty$, the electron and proton no longer interact, so the potential energy is simply 0. Thus to ionize hydrogen, you need to add 13.6 eV.
- The full quantum mechanical solution does not give simple orbits, but this model still does give reasonable results for the mean distance of the electron from the nucleus in the various energy states.

29.5 The Quantum Mechanical Hydrogen Atom Skip

A full solution must go beyond the cartoon planetary orbit model we used. We end up needing 4 numbers (known as four *quantum* numbers) to fully describe the atomic state:

- n : The principal quantum number. $n = 1, 2, 3, 4, \dots$. The energy is given by
$$E_n = -\frac{13.6 \text{ eV}}{n^2}.$$
- ℓ : Orbital quantum number. $\ell = 0, 1, \dots, n - 1$. The electron's orbital angular momentum has magnitude $L = \sqrt{\ell(\ell + 1)} \frac{h}{2\pi}$. Nomenclature:

ℓ	orbital label
0	s
1	p
2	d
3	f
4	g

29.5 The Quantum Mechanical Hydrogen Atom Skip

Quantum numbers, continued ...

- m_ℓ : magnetic quantum number: $m_\ell = -\ell, -\ell + 1, \dots, 0, \dots, \ell - 1, \ell$.
e.g. For $\ell = 1$, $m_\ell = -1, 0$, or 1 . For $\ell = 2$, $m_\ell = -2, -1, 0, 1$, or 2 . This gives the z -component of the angular momentum: $L_z = m_\ell \left(\frac{h}{2\pi} \right)$.
- m_s : electron “spin”: $+\frac{1}{2}$ or $-\frac{1}{2}$. (Although a classical picture of an electron as a spinning ball of charge proves unworkable, the name persists.) This term represents the z -component of the electron angular momentum $s_z = m_s \left(\frac{h}{2\pi} \right)$.

A complete description of the state of a hydrogen atom thus involves giving all four quantum numbers.

29.6 Multi-electron Atoms **Skip**

The same nomenclature and quantum numbers carry on to the description of atoms with more than one electron, even though we do not have precise analytical solutions for multielectron atoms. This section discusses how to assign electrons to these states and build up the periodic table. It is interesting reading, but will not be on the final exam.

29.7 Excited States and Spectra

This section goes into a bit more detail about how excited states are related to the spectra we observe. For the most part it is review. However, one important new idea is about *collisional excitation*. Consider an electron colliding with an atom in an initial state $E_{\text{atom},i}$. Conservation of energy requires that

$$E_{\text{electron},i} + E_{\text{atom},i} = E_{\text{electron},f} + E_{\text{atom},f}$$

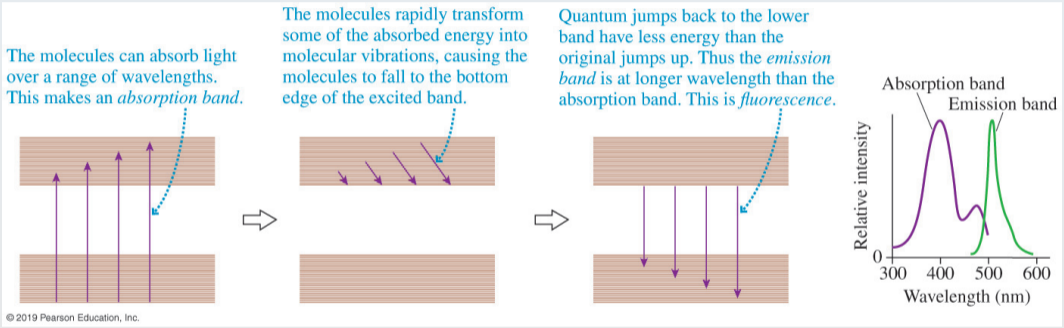
See example 29.9 in the text, and the Ch. 29 collision excitation example posted on Moodle.

- Emission spectra: Do not worry about detailed “selection” rules. We will only be concerned with energies.
- X-rays: Read. Don't worry about details. Basic idea is still $E_{\text{photon}} = \Delta E = \frac{hc}{\lambda}$.

29.8 Molecules **Skip**

- Energy is still quantized.
- Some of energy levels are *very* close together, effectively forming an almost-continuous band.
- Molecules can vibrate in many ways (as well as rotate); different vibration states correspond to different energy levels. Instead of emitting a photon, a molecule might change its vibration state.
- In thermal equilibrium, nearly all molecules are in the lowest energy levels.

Fluorescence Skip



Similar fluorescence was shown in the prelecture video.

29.9 Stimulated Emission and Lasers

You may skip this section. However, integrated example 29.10 is a good review of a number of related topics, and is worth reading.

What's Next?

- Chapter 30: Nuclear Physics
- Final review
- Final exam: Wednesday, May 11, 2022 @ 12:00 noon.