Problem 5: (20 pts.) Doppler ultrasound is a technique used to measure blood flow by bouncing high-frequency sound waves off circulating blood cells. It can be used to measure blood flows in arteries and veins even for slow flows on the order of mm/s. Suppose that an ultrasound signal with a frequency of exactly 6.000 MHz is used. The speed of sound in the tissue is 1540 m/s. This sound wave is absorbed by a red blood cell moving away from the detector at 3.4×10^{-2} m/s.

a. (10 pts.) What is the frequency of the sound as received by the blood cell?
Hints:

- You may leave your answer to this part in symbolic form.
- If you choose to plug in the numbers, be sure not to round off—keep at least 7 digits in your intermediate answer.

b. (10 pts.) The blood cell now re-radiates the sound at the frequency you found in part a. Since the blood cell is a moving source, the detector receives a doppler-shifted signal. What is the difference between the original 6.000 MHz frequency and the detected double-doppler-shifted reflected frequency?

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$f_{so} = 6.000\,MHz$ (original)

a. (10 pts.) What is the frequency of the sound as received by the blood cell?

Hints: $v = 1540\,m/s$ $v_{La} = 3.4 \times 10^{-2}\,m/s$ $v_{Sa} = 0$

- You may leave your answer to this part in symbolic form.
- If you choose to plug in the numbers, be sure not to round off—keep at least 7 digits in your intermediate answer.

$$f_{La} = \left(\frac{v - v_{La}}{v}\right) f_{so} = \left(1 - \frac{v_{La}}{v}\right) f_{so}$$

$$f_{La} = \left(1 - \frac{3.4 \times 10^{-2}\,m/s}{1540\,m/s}\right)(6.000 \times 10^{6}\,Hz)$$

$$f_{La} = 5\,999\,868\,Hz$$

b. (10 pts.) The blood cell now re-radiates the sound at the frequency you found in part a. Since the blood cell is a moving source, the detector receives a doppler-shifted signal. What is the difference between the original 6.000 MHz frequency and the detected double-doppler-shifted reflected frequency?

$f_{sb} = f_{La}$ $v_{sb} = 3.4 \times 10^{-2}\,m/s$ $v_{Lb} = 0$

$$f_{Lb} = \left(\frac{v}{v + v_{sb}}\right) f_{sb}$$

Numerically: $f_{Lb} = \left(\frac{1540}{1540 + 3.4 \times 10^{-2}}\right)(5\,999\,868\,Hz)$

$$f_{Lb} = 5\,999\,735\,Hz.$$

symbolically: put in result from (a)

$$f_{Lb} = \left(\frac{v}{v + v_{sb}}\right)\left(\frac{v - v_{La}}{v}\right) f_{so} = \left(\frac{v - v_{La}}{v + v_{sb}}\right) f_{so}$$

Then

$$\Delta f = f_{so} - f_{Lb} = \left[1 - \frac{v - v_{La}}{v + v_{sb}}\right] f_{so}$$

put over common denominator

$$\Delta f = \left[\frac{v + v_{sb} - (v - v_{La})}{v + v_{sb}}\right] f_{so}$$

$$\Delta f = \left[\frac{v_{sb} + v_{La}}{v + v_{sb}}\right] f_{so} .$$ Since

v_{sb} and v_{La} are both the speed of the red blood cell v_{RBC}

$$\Delta f = \left[\frac{2\, v_{RBC}}{v + v_{RBC}}\right] f_o$$

$$\Delta f = \frac{2 \cdot 3.4 \times 10^{-2}\ m/s}{1540 + 3.4 \times 10^{-2}\ m/s} \cdot \left(6.000 \times 10^{6}\ Hz\right)$$

$$\boxed{\Delta f = 265\ Hz}$$

Note since the v's cancel in the numerator, we don't need to take extra ordinary care to avoid roundoff errors.