

**21-26**

A 2.00-MHz sound wave travels through a pregnant woman's abdomen and is reflected from the fetal heart wall of her unborn baby. The heart wall is moving toward the sound receiver as the heart beats. The reflected sound is then mixed with the transmitted sound, and 85 beats per second are detected. The speed of sound in body tissue is 1500 m/s. Calculate the speed of the fetal heart wall at the instant this measurement is made.

$$f_0 = 2.00 \text{ MHz}$$

$$v = 1500 \text{ m/s}$$

$v_H = \text{speed of heart wall}$

1<sup>st</sup>: Heart is the "listener." It absorbs sound.

$$f_{L1} = \frac{v + v_{L1}}{v + v_{S1}} \cdot f_0 = \frac{v + v_H}{v} f_0$$

↑  
heart

0 → source is at rest

Use + sign because heart is approaching so frequency is higher.

2<sup>nd</sup> Heart re-radiates - it is now the source. The speed of the source is  $v_{S2} = v_H$ . The frequency of the source is what we found just above:  $f_{S2} = f_{L1}$ . The listener is the detector, which is at rest.

Use the - sign because the heart wall is approaching, which boosts the frequency.

$$f_{L2} = \frac{v + v_{L2}}{v - v_{S2}} f_{S2} = \frac{v}{v - v_H} \cdot f_{L1}$$

Plugging in the algebraic result above for  $f_{L2}$ :

$$f_{L2} = \left( \frac{v}{v - v_H} \right) \left( \frac{v + v_H}{v} \right) f_0$$

$$f_{L2} = \left( \frac{v + v_H}{v - v_H} \right) f_0.$$

Finally, we are given the difference between  $f_{L2}$  and  $f_0$  is  $\Delta f = 85 \text{ Hz}$ .

$$\Delta f = f_{L2} - f_0 = \left( \frac{v + v_H}{v - v_H} - 1 \right) f_0$$

put over a common denominator:  $\Delta f = \left( \frac{v + v_H - (v - v_H)}{v - v_H} \right) f_0$

$$\Delta f = \left( \frac{2v_H}{v - v_H} \right) \cdot f_0$$

Solve for  $v_H$ . The algebra is messy, but we can approximate since  $v \gg v_H$ . Recall  $v = 1500 \text{ m/s}$ .

$$\Delta f \approx \frac{2v_H}{v} \cdot f_0 \Rightarrow v_H \approx \frac{1}{2} \frac{\Delta f}{f_0} \cdot v$$

Plugging in

$$v_H \approx \frac{1}{2} \frac{(85 \text{ Hz})}{(2.00 \times 10^6 \text{ Hz})} \cdot 1500 \text{ m/s}$$

$$v_H = 0.032 \text{ m/s}$$

The problem doesn't ask for it, but working backwards:

$$f_0 = 2,000,000 \text{ Hz}$$

$$f_{L1} = \frac{v + v_H}{v} f_0 \approx 2000042 \text{ Hz}$$

$$f_{L2} = \frac{v}{v - v_H} f_{L1} \approx 2000085 \text{ Hz}$$

Measuring absolute frequencies to 7 digits is quite hard. Measuring beats,  $\Delta f = 85 \text{ Hz}$ , is relatively easy.