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Physics 112
Chapter 14 Notes—Review
Simple Harmonic Motion

14 Oscillations

14.1 Equilibrium and Oscillation

Often in nature, we observe systems that oscillate when they are slightly displaced from equilibrium.

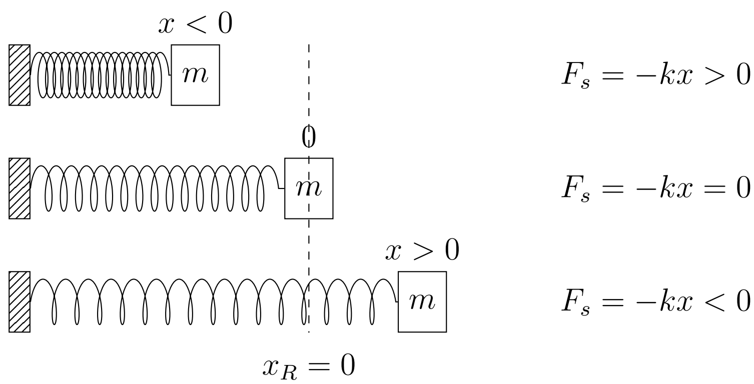
14.2 Linear Restoring Forces and Simple Harmonic Motion

The model system we consider is a mass on a horizontal spring.

If the relaxed position of the spring is taken to be $x_R = 0$, then the spring force is given by Hooke's Law:

$$F_s = -kx \tag{1}$$

where k is a positive constant known as the spring constant (in Newtons/meter).



14.3 Describing Simple Harmonic Motion

A particle undergoing simple harmonic motion oscillating with amplitude A goes back and forth between $-A$ and $+A$. The time to complete one complete cycle is the period T. If the particle starts at rest at the maximum, *i.e.* $x = +A$, then the motion is described by

$$x(t) = A\cos\left(\frac{2\pi}{T}t\right)$$

In addition to the period T, we often describe the motion in terms of the frequency $f = 1/T$ or the angular frequency $\omega = 2\pi f$. These are related by

$$\omega = 2\pi f = \frac{2\pi}{T}$$

so that the motion can be written in any of several equivalent ways:

$$x(t) = A\cos\left(\frac{2\pi}{T}t\right)$$
$$x(t) = A\cos(2\pi f t)$$
$$x(t) = A\cos(\omega t)$$

The amplitude A is set by the initial conditions. The frequency is set by the physics of what's oscillating. For the mass on a spring,

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

The velocity is $v = \frac{dx}{dt}$

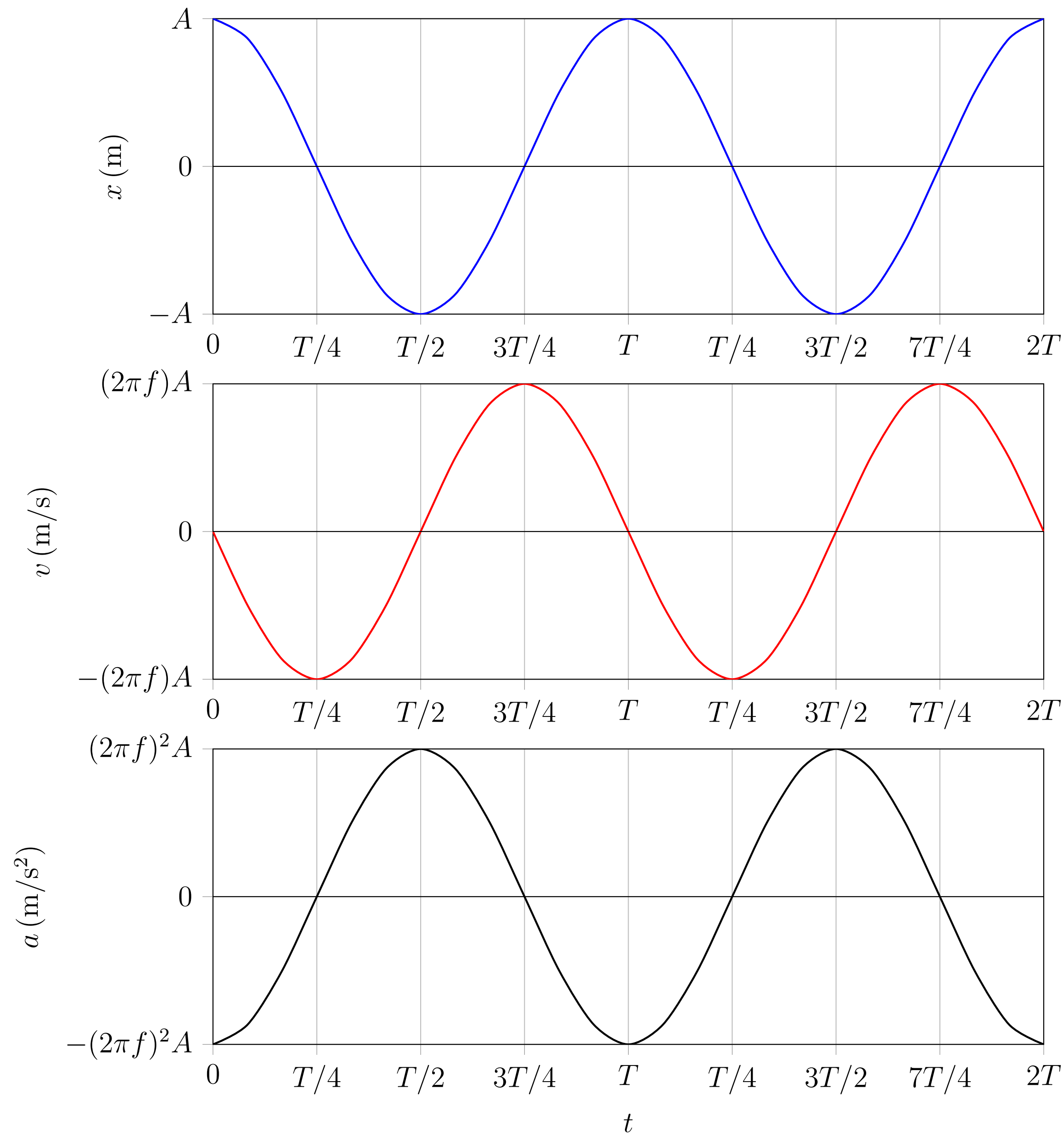
$$v(t) = -(2\pi f)A\sin(2\pi f t)$$

The velocity has a maximum amplitude $(2\pi f)A$ when the particle passes through the equilbrium point, and zero at the extremes of the motion.

Finally, the acceleration is given either by taking the derivative $\frac{dv}{dT}$, or by using Newton's second law:

$$ma = F$$
$$ma = -kx$$
$$a = -\frac{k}{m}x = -(2\pi f)^2 A\cos(2\pi f t)$$

The acceleration has a maximum amplitude $(2\pi f)^2 A$ when the spring is stretched or compressed the most, at the extremes of the motion.



Some useful trigonometry reminders

A few trigonometry values will be particularly useful this semester. It will often be useful to quickly recognize whether a trig function is 0 or ± 1. Some handy results are shown in the following tables

$$\cos(0) = 1 \qquad \cos(\pi) = -1 \qquad \cos(2\pi) = 1$$
$$\cos\left(\frac{\pi}{2}\right) = 0 \qquad \cos\left(\frac{3\pi}{2}\right) = 0$$
$$\cos(-x) = \cos(x)$$

$$\sin(0) = 0 \qquad \sin(\pi) = 0 \qquad \sin(2\pi) = 0$$
$$\sin\left(\frac{\pi}{2}\right) = 1 \qquad \sin\left(\frac{3\pi}{2}\right) = -1$$
$$\sin(-x) = -\sin(x)$$

14.4 Energy in Simple Harmonic Motion

Many oscillation problems can be solved using $x(t) = A\cos(2\pi ft)$, but since the spring force is conservative, it is often easier to use energy conservation.

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

This is true at all times, but is easiest to evaluate either at the maximum amplitude, where the velocity is zero, or at the equilbrium position, where the velocity is maximum and the position is zero.

$$E = K + U = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}m(2\pi fA)^2 = \frac{1}{2}kA^2$$

The energy is proportional to amplitude squared and to frequency squared. Energy will be a useful tool for studying waves, and that amplitude and frequency scaling will appear again.