Physics 112

Chapter 15: Traveling Waves and Sound

Class Notes—Part 1

15 Traveling Waves and Sound

15.1 The Wave Model

In Phys 111, we mostly dealt with the motion of individual particles. Starting in this chapter, we adopt a different perspective: Waves describe a *collective* motion.

Much of physics is actually *wave* physics, such as optics, acoustics, and quantum mechanics.

A wave is a motion of a disturbance. There are a number of different types of waves (*e.g.* water, sound, and light waves, but there are also some common themes and terms that are helpful to understand. The textbook and pre-lecture videos do a good job introducing these ideas and terms.

Mechanical Waves

Mechanical waves involve the motion of a medium. Examples include waves on a string, sound and ultrasound waves, and water waves. It is important to not confuse the motion of the individual particles with the motion of the wave.

Electromagnetic Waves

Electromagnetic waves are waves in the electromagnetic field. (More on that later this semester.) They do not require a medium—they can travel through a vacuum—though they also can travel through a medium. Quantum mechanical probability amplitudes can also be described as waves. (More on that in Chapter 28.)

Transverse Waves

In *transverse* waves, the motion of particles is *perpendicular* to the motion of the wave. Waves on a vibrating string are transverse, as are electromagnetic waves.

Longitudinal Waves

In *longitudinal* waves, the motion of particles is *parallel* to the motion of the wave. Sound waves are longitudinal waves.

Waves on a Slinky can be longitudinal or transverse. Earthquakes typically have both transverse and longitudinal waves that travel at different speeds. Water waves typically involve a combination of both transverse and longitudinal motion.

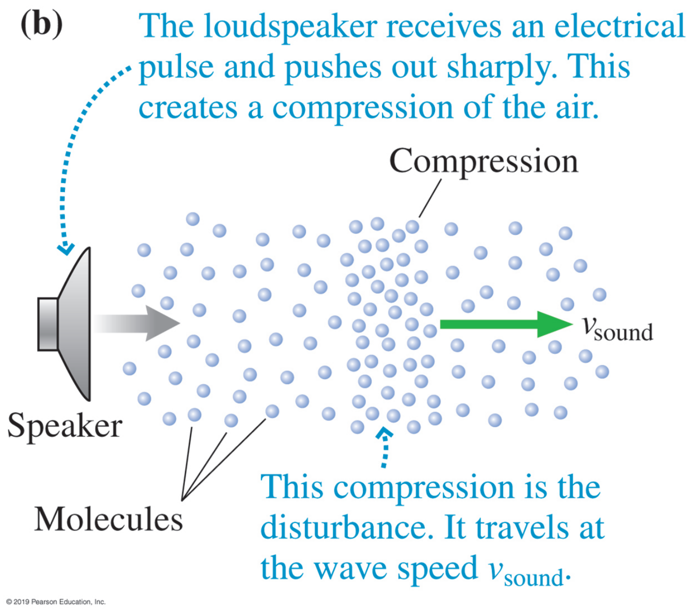

Figure 1: Sound waves (Figure 15.4 (b))

15.2 Traveling Waves

Consider the motion of a pulse traveling to the right at a speed v:

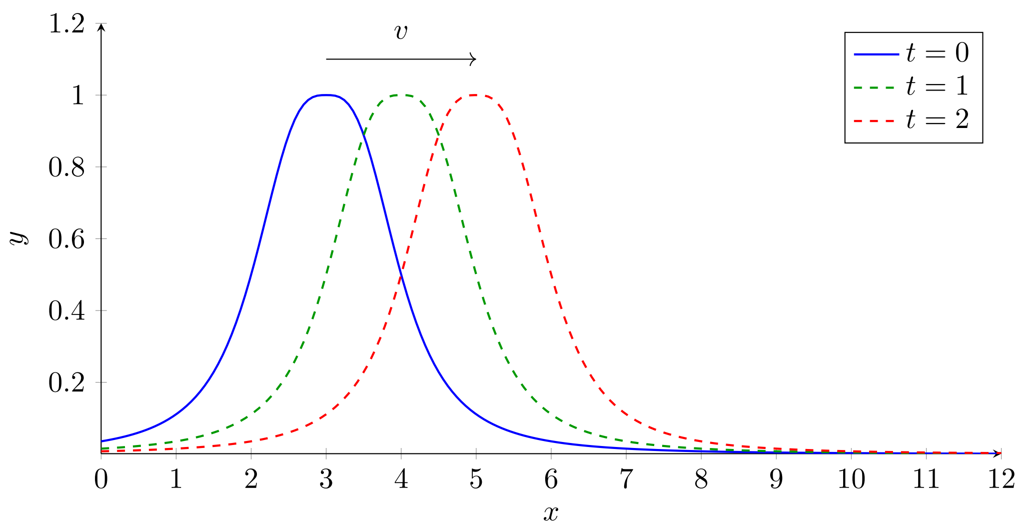

Figure 2: A traveling pulse moves with speed v.

The shape of the pulse is $f(x)$. If it moves to the *right* with speed v, then the function f is of the form $f(x - vt)$. That is, the pulse has the same shape, but all the x coordinates are shifted to the right a distance vt. If it moves to the *left* with speed v, then the function is of the form $f(x + vt)$.

Wave Speed is a Property of the Medium

The wave speed v is set by the physics of what is waving. For waves on a string, it depends on the tension and on the density of the string. We typically define a *linear mass density* μ by

$$\mu = \frac{m}{L}$$

where m is the total mass of the string, and L is the total length. The linear mass density μ (expressed in kg/m) is a *local* property of the medium. Using a longer string (which makes both L and M larger) doesn't affect the wave speed.

The wave speed v is set by the physics of what is waving.

- Waves on a string under tension F_T:

$$v = \sqrt{\frac{F_T}{\mu}}$$

- Example: Consider a violin string of length $L = 0.327\,\text{m}$ and mass $M = 2.80 \times 10^{-4}\,\text{kg}$. Assume the string is under tension $F_T = 70.0\,\text{N}$. What is the wave speed?

  First, compute the linear mass density:

$$\mu = \frac{M}{L} = \frac{2.80 \times 10^{-4}\,\text{kg}}{0.327\,\text{m}} = 8.56 \times 10^{-4}\,\text{kg/m}$$

  The wave speed is then:

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{70.0\,\text{N}}{8.56 \times 10^{-4}\,\text{kg/m}}} = 286\,\text{m/s}$$

- Sound waves

$$v = \sqrt{\frac{\gamma R T}{M}}$$

  where $R = 8.314\,\text{J/mol} \cdot \text{K}$, T is the temperature in Kelvin, M is the molar mass (kilograms for one mole), and γ is a constant equal to the ratio of specific heat capacities:

$$\gamma = \frac{C_p}{C_V}$$

  and is 1.67 for a monatomic ideal gas, and about 1.40 for nitrogen and oxygen. We will use a standard value of

$$v_{\text{sound}} = 343\,\text{m/s}$$

  for the speed of sound in air at normal temperatures.

- General form: The general idea for a mechanical system is that

$$v \propto \sqrt{\frac{\text{measure of the restoring force}}{\text{measure of inertia}}}$$

- Electromagnetic waves in a vacuum (*e.g.* light, radio, X-rays):

$$v = c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \approx 2.998 \times 10^8\,\text{m/s}$$

Table 1: The speed of sound in various media

Medium	**Speed (m/s)**
Air (20 °C)	343
Helium (0 °C)	970
Water	1480
Aluminum	5100
Lead	1200
Diamond	12 000

15.3 Mathematical Description of a Wave

The simplest wave to consider is a traveling sine wave.

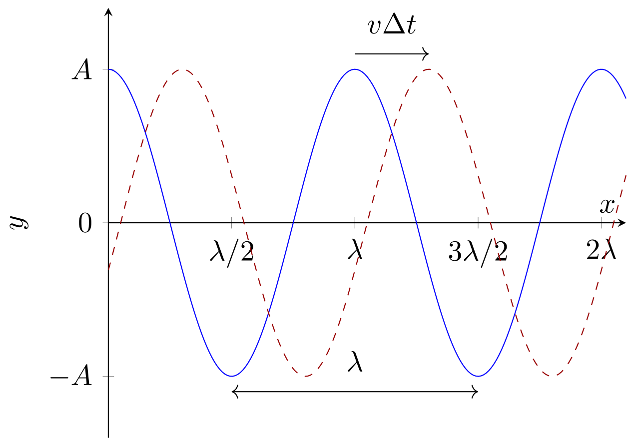

Figure 3: A traveling sinusoidal wave moving to the right with speed v. The wave is shown at times t and $t + \Delta t$.

- Amplitude = A
- Wave speed = v
- Wavelength = λ = distance to repeat (at fixed time)
- Period = T = time to repeat (at fixed position)

If you sit at a fixed x, how long do you wait between peaks? The next peak is a distance λ away, and is moving at speed v.

$$\lambda = vT \implies T = \frac{\lambda}{v}$$

The initial wave shape at time zero is

$$f(x) = A\cos\left(\frac{2\pi}{\lambda}x\right) .$$

It oscillates between $-A$ and $+A$, and repeats at a distance of λ. (The argument to the cos function is in radians.) To make it a traveling wave, we simply replace x by $x - vt$. The equation for the traveling sinusoidal wave is thus

$$y(x,t) = A\cos\left(\frac{2\pi}{\lambda}(x - vt)\right)$$
$$y(x,t) = A\cos\left(2\pi\left(\frac{x}{\lambda} - \frac{v}{\lambda}t\right)\right)$$
$$y(x,t) = A\cos\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right)$$

where we have used the result above that $v/\lambda = 1/T$.

Recall from Ch. 14 that there were multiple useful ways to express the oscillation period or frequency:

$$\omega = 2\pi f = \frac{2\pi}{T},$$

where f is the frequency, and ω is the angular frequency. In a similar vein, we also sometimes define a "wavenumber" $k = \frac{2\pi}{\lambda}$. Accordingly, you will often see the sinusoidal wave written in a variety of equivalent forms:

$$y(x,t) = A\cos\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right)$$
$$y(x,t) = A\cos\left(2\pi\left(\frac{x}{\lambda} - ft\right)\right)$$
$$y(x,t) = A\cos\left(\frac{2\pi}{\lambda}x - 2\pi ft\right)$$
$$y(x,t) = A\cos(kx - \omega t)$$

A number of end-of-chapter problems focus on interpreting and converting among these various forms.

Key Relation: For a wave, you typically choose the frequency when you excite the wave. The wave speed is determined by the physical properties of what's waving. The wavelength is then determined by the equation:

$$v = \frac{\lambda}{T} = \lambda f$$

This relation $\boxed{v = \lambda f}$ holds for all sinusoidal waves, both mechanical and electromagnetic.

Thus yet another way of writing the equation for a wave is obtained by factoring out $\frac{2\pi}{\lambda}$ as follows:

$$y(x,t) = A\cos\left(\frac{2\pi}{\lambda}x - 2\pi ft\right) = A\cos\left(\frac{2\pi}{\lambda}(x - \lambda ft)\right)$$
$$y(x,t) = A\cos\left(\frac{2\pi}{\lambda}(x - vt)\right)$$

which is indeed of the form $f(x - vt)$ discussed above.