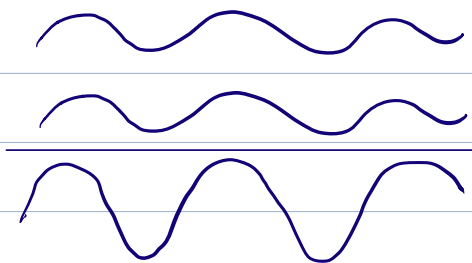


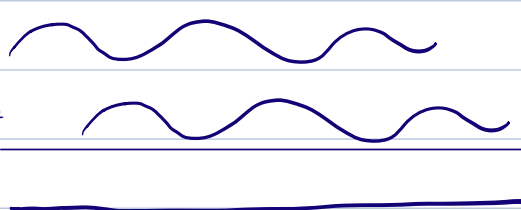
16.6 Interference of Waves from Two Sources

Constructive
Interference: $+$



The diagram shows two waves in phase, one above the other. A plus sign is between them. Below them is a horizontal line, and below that is a single wave with double the amplitude of the individual waves, representing constructive interference.

Destructive
Interference: $+$



The diagram shows two waves out of phase, one above the other. A plus sign is between them. Below them is a horizontal line, and below that is a single flat line, representing destructive interference.

(i.e. flat - nothing - waves cancel.)

Consider: do the peaks line up / reinforce?
What is the difference? Destructive is
shifted by $\frac{1}{2} \lambda$ (or $\frac{3}{2} \lambda$, or $\frac{5}{2} \lambda$, etc.).

Why might waves be shifted? Many possibilities. One simple one: if the waves travel different distances, e.g.

$$y_1(x, t) = A \cos \left(2\pi \left(\frac{x_1}{\lambda} + \frac{t}{T} \right) \right)$$

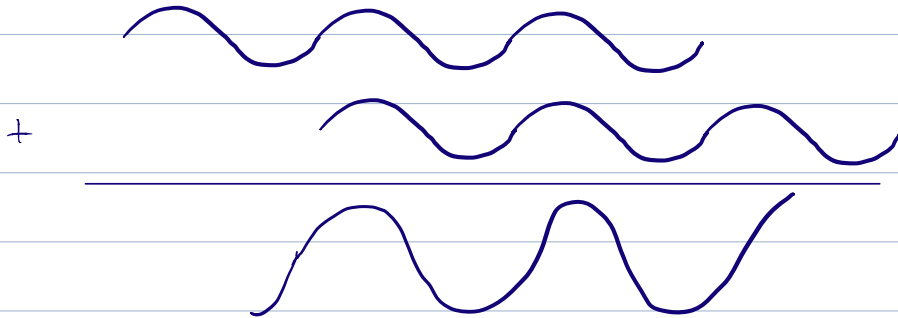
$$y_2(x, t) = A \cos \left(2\pi \left(\frac{x_1 + \Delta x}{\lambda} + \frac{t}{T} \right) \right)$$

Mathematically, the phase difference here is

$$\phi = \frac{2\pi \Delta x}{\lambda}$$

Constructive

If $\Delta x = 0, \lambda, 2\lambda, 3\lambda, \dots$, then
the waves line up and interfere constructively



constructive: $\Delta x = m\lambda$, where
 $m = 0, \pm 1, \pm 2, \dots$ an integer

Mathematically:

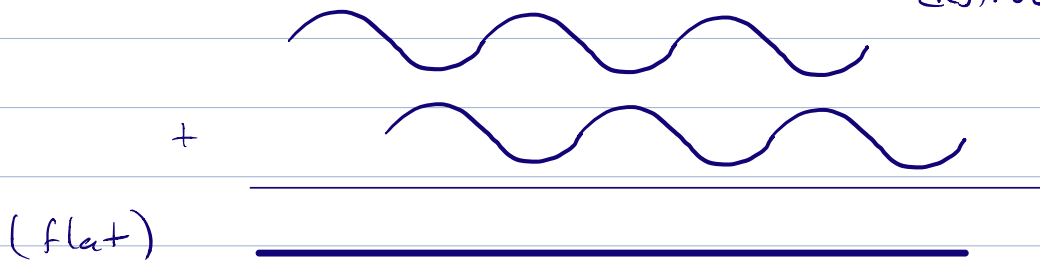
$$\text{constructive: } \phi = \frac{2\pi\Delta x}{\lambda} = \frac{2\pi m\lambda}{\lambda} = m(2\pi)$$

where $m = \text{an integer}, 0, \pm 1, \pm 2, \dots$

Destructive

If $\Delta x = \frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda, \dots$

the waves line up out of phase, and interfere destructively



destructive: $\Delta x = (m + \frac{1}{2})\lambda$, where
 $m = 0, \pm 1, \pm 2, \dots$ an integer

Mathematically = destructive:

$$\phi = \frac{2\pi\Delta x}{\lambda} = \frac{2\pi(m + \frac{1}{2})\lambda}{\lambda} = m(2\pi) + \pi$$

where $m = \text{an integer}, 0, \pm 1, \pm 2, \dots$

Constructive: waves are "in phase"

Destructive: waves are "out of phase"
by $\frac{1}{2}\lambda$ or by π radians.

This works even if both waves don't travel along the x -axis, as long as they start out in phase.

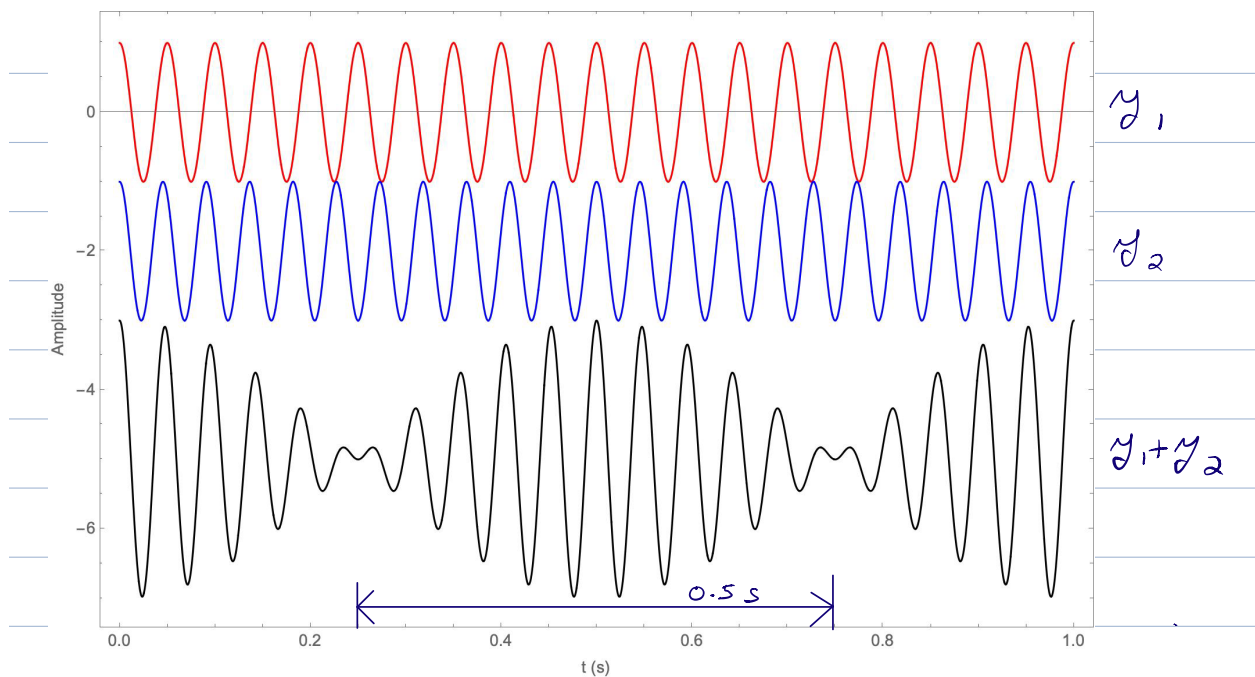
Constructive: $\Delta r = m\lambda$

Destructive: $\Delta r = (m + \frac{1}{2})\lambda$

$m = 0, \pm 1, \pm 2, \dots$

16.7 Beats

Two sources with different frequencies.



$$f_1 = 20\text{Hz} \quad f_2 = 22\text{Hz} \quad \Delta f = 2\text{Hz}$$

Net result: Amplitude oscillates with a
"Beat" frequency $f_{\text{beat}} = |f_2 - f_1|$.

This is useful for detecting small differences. Recall Doppler example:

$$f_{\text{orig}} = 6,000,000 \text{ Hz}$$

$$f_{\text{reflected}} = 5,999,735 \text{ Hz}$$

If you combine the signals, the beat frequency $f_{\text{beat}} = |f_{\text{orig}} - f_{\text{reflected}}| = 265 \text{ Hz}$ is easy to detect and measure accurately.