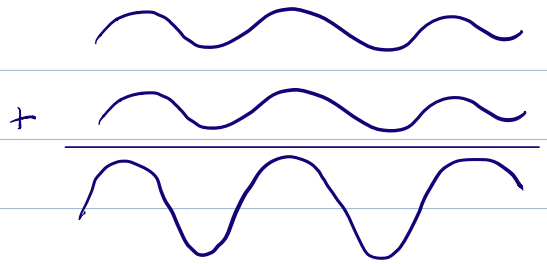
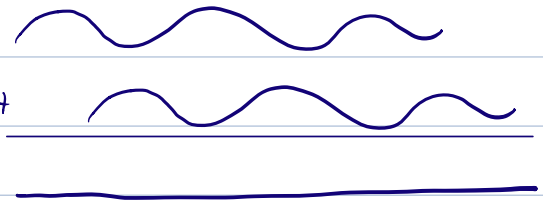
Constructive Interference:



Destructive Interference:



(i.e. flat - nothing — waves cancel.)

Consider: do the peaks line up / reinforce?
What is the difference? Destructive is shifted by $\frac{1}{2}\lambda$ (or $\frac{3}{2}\lambda$, or $\frac{5}{2}\lambda$, etc.).

Why might waves be shifted? Many possibilities. One simple one: if the waves travel different distances, e.g.

$$y_1(x,t) = A\cos\left(2\pi\left(\frac{x_1}{\lambda} + \frac{t}{T}\right)\right)$$

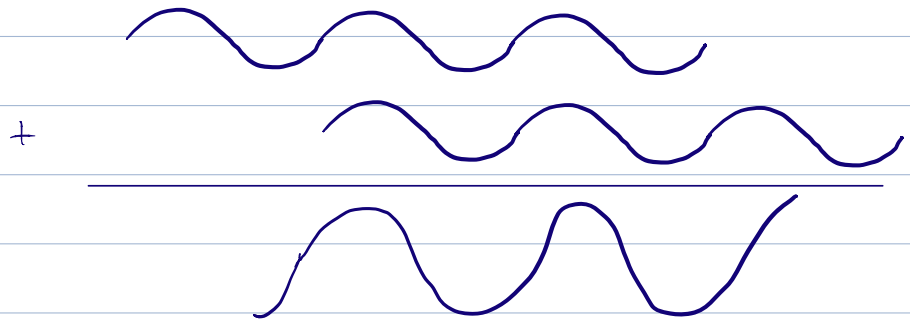
$$y_2(x,t) = A\cos\left(2\pi\left(\frac{x_1 + \Delta x}{\lambda} + \frac{t}{T}\right)\right)$$

Mathematically, the phase difference here is

$$\phi = \frac{2\pi\Delta x}{\lambda}$$

Constructive

If $\Delta x = 0, 1\lambda, 2\lambda, 3\lambda, \ldots$, then the waves line up and interfere constructively



constructive: $\Delta x = m\lambda$, where

$m = 0, \pm 1, \pm 2, \ldots$ an integer

Mathematically:

constructive:

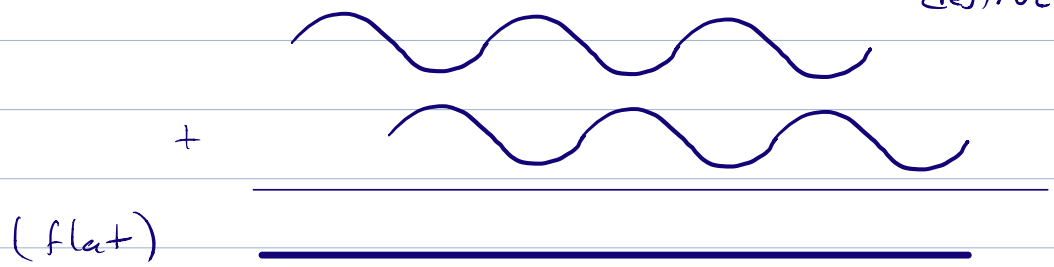
$$\phi = \frac{2\pi\Delta x}{\lambda} = \frac{2\pi m\lambda}{\lambda} = m(2\pi)$$

where m = an integer, $0, \pm 1, \pm 2, \ldots$

Destructive

If $\Delta x = \frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda, \ldots$

the waves line up out of phase, and interfere destructively



destructive: $\Delta x = \left(m + \frac{1}{2}\right)\lambda$, where

$m = 0, \pm 1, \pm 2, \ldots$ an integer

Mathematically = destructive:

$$\phi = \frac{2\pi \Delta x}{\lambda} = \frac{2\pi\left(m + \frac{1}{2}\right)\lambda}{\lambda} = m(2\pi) + \pi$$

where m = an integer, $0, \pm 1, \pm 2, \ldots$.

Constructive: waves are "in phase"

Destructive: waves are "out of phase" by $\frac{1}{2}\lambda$ or by π radians.

This works even if both waves don't travel along the x-axis, as long as they start out in phase.

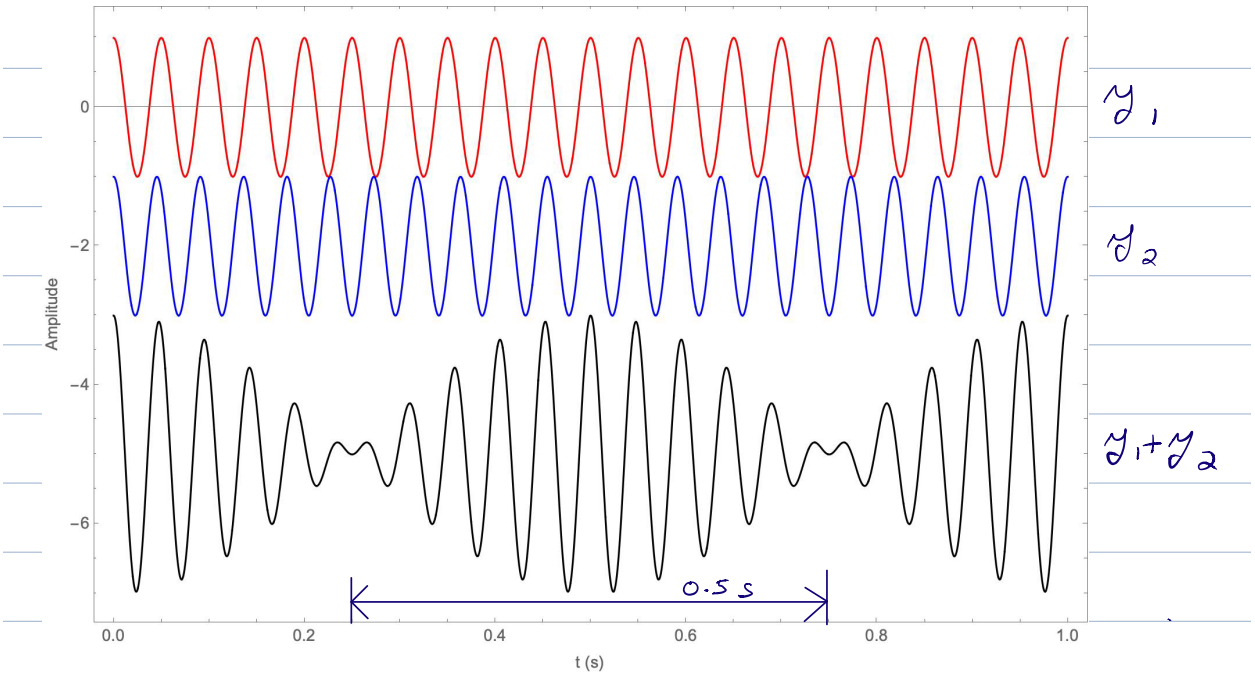
Constructive: $\Delta r = m\lambda$

Destructive: $\Delta r = \left(m + \frac{1}{2}\right)\lambda$

$m = 0, \pm 1, \pm 2, \ldots$

16.7 Beats

Two sources with different frequencies.



$f_1 = 20Hz \qquad f_2 = 22Hz \qquad \Delta f = 2Hz$

Net result: Amplitude oscillates with a "Beat" frequency $f_{beat} = |f_2 - f_1|$.

This is useful for detecting small differences. Recall Doppler example:

$f_{orig} = 6,000,000$ Hz

$f_{reflected} = 5,999,735$ Hz

If you combine the signals, the beat frequency $f_{beat} = |f_{orig} - f_{reflected}| = 265$ Hz is easy to detect and measure accurately.