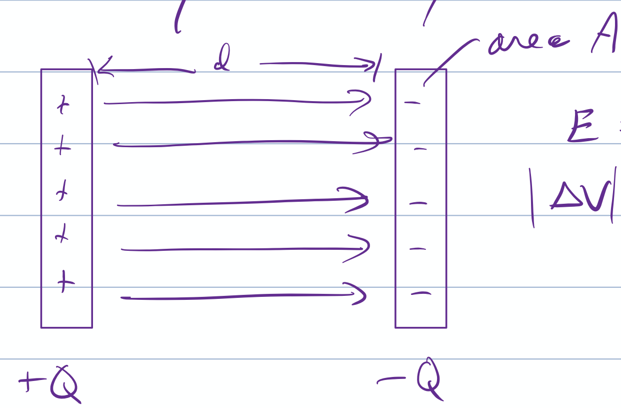
Capacitor: Two conductors separated by an insultor.

e.g. 2 parallel plates:



$$E = \frac{\sigma}{\epsilon_0} = \frac{Q/A}{\epsilon_0}$$

$$|\Delta V| = |Ed|.$$

Question: for a given potential difference ΔV, how much charge can you store?

$$E d = \Delta V$$

$$\frac{Q/A}{\epsilon_0} d = \Delta V$$

$$Q = \left(\frac{\epsilon_0 A}{d}\right) \Delta V \equiv C(\Delta V)$$

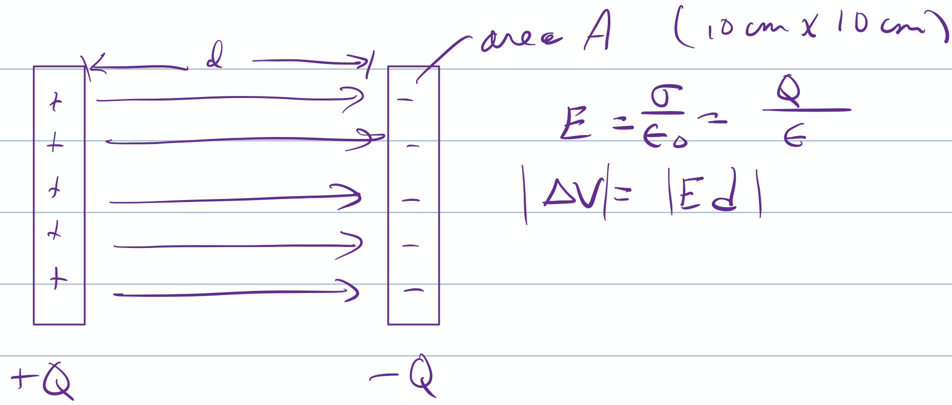
$C \equiv$ capacitance

unit $= \frac{\text{Coulomb}}{\text{Volt}} \equiv$ Farad $= F$

common units: $pF = 10^{-12} F$

$\mu F = 10^{-6} F$

e.g. our favorite parallel plates



$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon}$$

$$|\Delta V| = |Ed|$$

Let $A = (0.1m)^2 = 0.01m^2$

$d = 5mm = 0.005m$

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85\times10^{-12}\ C^2/Nm^2)(0.01m^2)}{0.005m}$$

$$C = 1.77\times10^{-11}\ F$$

If you chare it up to $+9V$, how much charge is stored?

$$Q = C(\Delta V) = (1.77\times10^{-11}F)(9.0V) = 1.59\times10^{-10}\ C$$

What is the electric field between the plates?

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q/A}{\epsilon_0} = \frac{(1.59\times10^{-10}C)/0.01m^2}{8.85\times10^{-12}\ C/Nm^2}$$

$$E = 1800\ N/C$$

OR $$E = \frac{\Delta V}{d} = \frac{9.0\ V}{0.005m} = 1800\ \frac{V}{m}$$

Dielectrics: Not on test

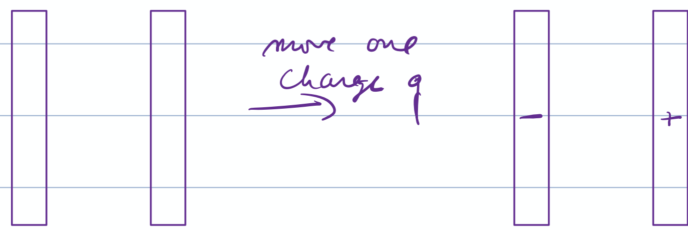
- enables storing more charge by reducing the electric field, and increasing the energy density.

21.8 energy and capacitors

Key idea: a charged capacitor stores energy.

Cartoon:

start

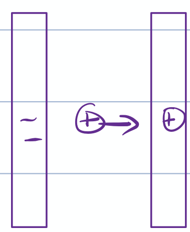


unchanged.

leaves net $-q$

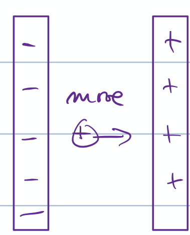
add $+$

add a second charge



This takes more work since the charge is attracted to the $-$ plate and repelled from the $+$ plate.

later...



takes even more work.

total work done = ener stored up.

$$U_c = \frac{1}{2}Q\left(\Delta V\right) = \frac{1}{2}C\left(\Delta V\right)^2 = \frac{1}{2}\frac{Q^2}{C}$$

(all related by $Q = C\Delta V$.)

e.g. our parallel plate capacitor above

$\Delta V = 9.0\ V \qquad C = 1.77 \times 10^{-11} F$

$U_C = \frac{1}{2} C (\Delta V)^2 = 7.17 \times 10^{-10} J$

tiny!

Improvements: use dielectrics to increase C.

$$C = \frac{\overbrace{K \epsilon_0 A}^{\text{dielectric constant (depends on material)}}}{\underset{\uparrow}{d}}$$

thickness – make very thin!

This turns out to be a very useful short-term energy storage device.

e.g. Ch21-capacitor-1.pdf.

Confusion alert! We now have 2 similar looking equations

$$\Delta U = q_0 (\Delta V)$$

$$U_c = \frac{1}{2} Q (\Delta V)$$

Why does one have 1/2 but the other doesn't?

$$\Delta U = q_0 \underbrace{(\Delta V)}_{\text{due to other charges, not } q_0}$$

$$U_c = \frac{1}{2} Q \underbrace{(\Delta V)}_{\text{due to the same charges } Q}$$

Electric field energy

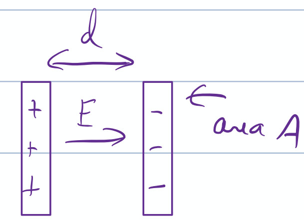
Where is the energy stored? It's a bit of a sloppy question – it is stored in the whole system. Still, it's useful to think of it as being stored in the electric field!

$$U_C = \frac{1}{2} Q (\Delta V)$$

$$= \frac{1}{2} C (\Delta V)^2$$

$$= \frac{1}{2} \frac{\epsilon_0 A}{d} \cdot (E d)^2$$

$$= \frac{1}{2} \epsilon_0 E^2 \underbrace{(A \cdot d)}_{\text{volume}}$$



$$U_C = \frac{1}{2} \epsilon_0 E^2 (\text{volume})$$

Electrical energy density

$$u_e \equiv \frac{U}{\text{volume}} = \frac{1}{2} \epsilon_0 E^2 \quad \left(\frac{\text{Joules}}{m^3}\right)$$

Later ...

magnetic fields also store energy

Electromagnetic waves transport energy