23.1 Circuit Elements and Diagrams

Read, and watch the pre-lecture video. We will encounter these issues as we discuss specific circuits.

23.2 Kirchhoff's Laws

Kirchhoff's Voltage Law (KVL): The sum of voltage drops around a closed loop = 0. This is really a conservation of energy statement. When a charge completes a loop around a circuit, it ends up back where it started.

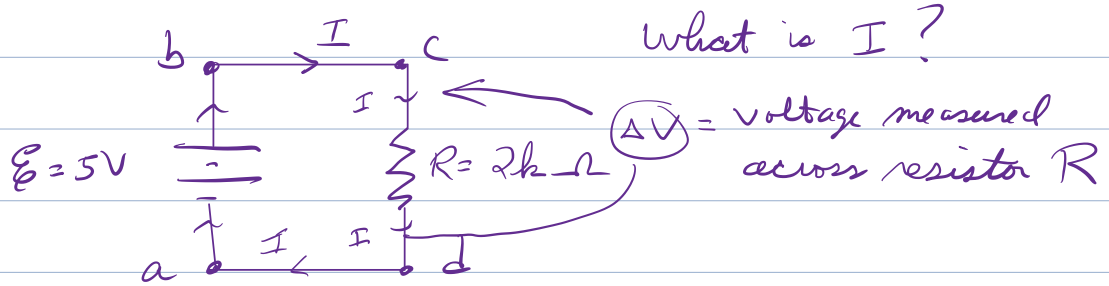
Kirchhoff's Current Law (KCL): The sum of currents into a junction = the sum of currents leaving a junction. This is really a statement of conservation of charge. There is no build-up of charge at a junction.

These are best illustrated through applications.

23.3 Series and Parallel

Start with a simple circuit.

What is I?



$$\text{KVL: } a \rightarrow b \rightarrow c \rightarrow d \rightarrow a = 0 \text{ Volts}$$

$$+\underset{\substack{\uparrow \\ \text{gain in battery}}}{\mathcal{E}} + 0 - \underset{\substack{\uparrow \\ \text{lose in resistor}}}{\Delta V} + 0 = 0$$

$$\Delta V = \mathcal{E} = 5\text{ V}$$

Apply Ohm's Law to the resistor

$$\Delta V = IR$$

$$I = \frac{\Delta V}{R} = \frac{5.0\text{V}}{2\text{k}\Omega} = 2.5\text{mA}$$

I is the same everywhere in this circuit.

Note on units:

$1\,k\Omega = 10^3\,\Omega$

$1\,mA = 10^{-3}\,A$

$(1\,k\Omega)(1\,mA) = 10^3\,\Omega \cdot 10^{-3}\,A =$

$= 1\,\Omega \cdot A = 1\,V.$

Power considerations:

Power supplied by the battery

$P_{\mathcal{E}} = I\mathcal{E} = (2.5\,mA)(5V) = 12.5\,mW$

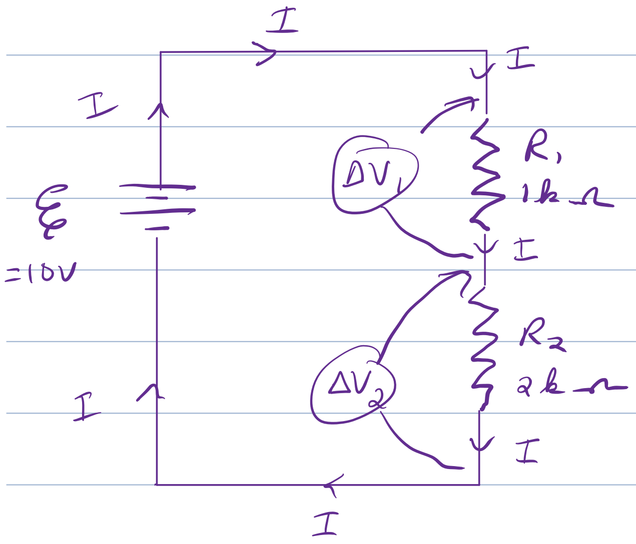
Power dissipated by the resistor

$P_R = I^2 R = (2.5\,mA)^2 (2\,k\Omega) = 12.5\,mW$

They match!

Series Example

$\mathcal{E} = 10V \quad R_1 = 1k\Omega \quad R_2 = 2k\Omega$



Series: same current goes through first one then the other

Question: what is I ?

(Poll on I & V.)

KVL $\quad \mathcal{E} - \Delta V_1 - \Delta V_2 = 0$

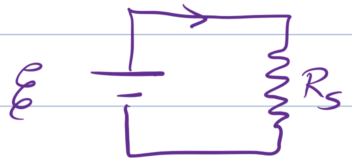
$$\mathcal{E} - IR_1 - IR_2 = 0$$

$$\mathcal{E} = I(R_1 + R_2)$$

$$I = \frac{\mathcal{E}}{(R_1 + R\ )} = \frac{10V}{1k\Omega + 2k\Omega} = \quad 3.33\,mA$$

Series equivalent: As far as the battery is concerned, this series combination acts as if the series were replaced by a single resistor

$$R_S = R_1 + R_2 = 3k\Omega$$



This circuit has the same

Generalized Series equivalent:

$$R_s = R_1 + R_2 + R_3 + \ldots$$

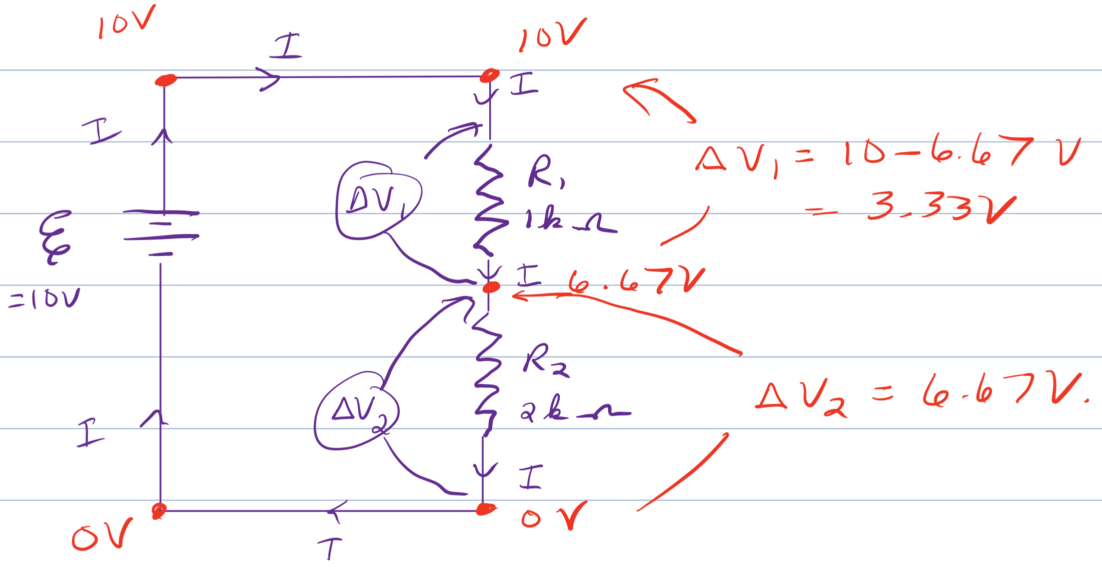
Go back and label voltages:

$\Delta V_1 = IR_1 = (3.33\,mA)(1\,k\Omega) = 3.33\,V$

$\Delta V_2 = IR_2 = (3.33\,mA)(2\,k\Omega) = 6.67\,V$

add: $\Delta V_1 + V_2 = 10\,V$ ✓.

also – label on diagram.



Power considerations

Battery : $P_{\mathcal{E}} = \mathcal{E} I = (10V)(3.33mA) = 33.3 mW$

Resistor 1 :

$P_1 = I^2 R_1 = (3.33mA)^2 (1k\Omega) = 11.1 mW$

(OR : $P_1 = I \Delta V_1 = (3.33mA)(3.33V) = 11.1mW$)

Resistor 2 :

$P_2 = I^2 R_2 = (3.33mA)^2 (2k\Omega) = 22.2 mW$

(OR : $P_2 = I \Delta V_2 = (3.33mA)(6.67V) = 22.2 mW$)

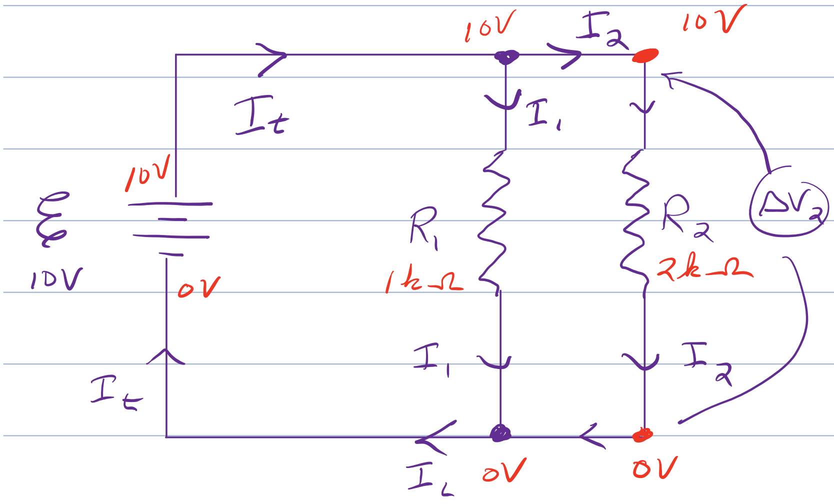
Total $P_1 + P_2 = 11.1 mW + 22.2 mW = 33.3 mW$ ✓

Parallel example

$\mathcal{E} = 10\text{ V}$

$R_1 = 1\text{k}\Omega$

$R_2 = 2\text{k}\Omega$



Parallel: current goes through one or the other, but not both.

Question: what is I_t? (Poll on I & V)

KCL: at junction: $I_t = I_1 + I_2$.

What are I_1 and I_2? Use Ohm's Law

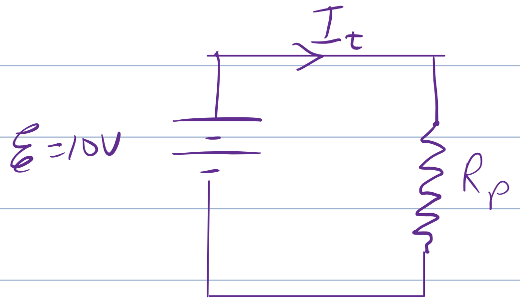
$$I_1 = \frac{\Delta V_1}{R_1} = \frac{\mathcal{E}}{R_1} = \frac{10\text{V}}{1\text{k}\Omega} = 10\text{ mA}$$

$1\text{mA} \cdot 1\text{k}\Omega = 1\text{V}$

$$I_2 = \frac{\Delta V_2}{R_2} = \frac{\mathcal{E}}{R_2} = \frac{10\text{V}}{2\text{k}\Omega} = 5\text{mA}$$

$$I_t = I_1 + I_2 = 10\text{mA} + 5\text{mA} = 15\text{mA}$$

From the perspective of the battery, this is equivalent to a single resistor R_p



$$I_t = \frac{\mathcal{E}}{R_p} \Rightarrow R_p = \frac{\mathcal{E}}{I_t}$$

$$R_p = \frac{10\text{V}}{15\text{mA}} = 0.667\text{k}\Omega$$

symbolically: use KCL

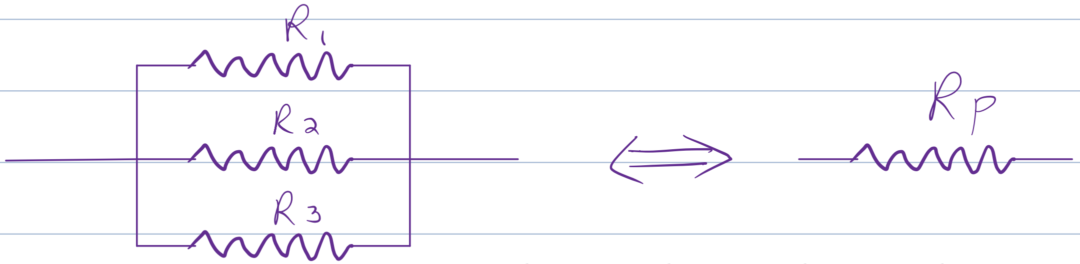
$$I_t = I_1 + I_2$$

$$\frac{\mathcal{E}}{R_p} = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2}$$

$$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}}$$

$$\underline{R_p = \frac{R_1 R_2}{R_1 + R_2}}$$

Generalized parallel resistance



$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

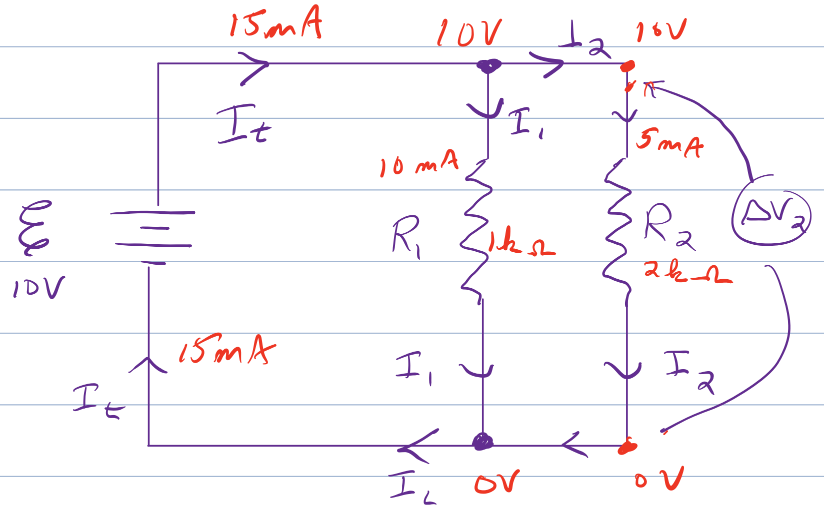
e.g. here $\frac{1}{R_p} = \frac{1}{\underline{1 k\Omega}} + \frac{1}{\underline{2 k\Omega}} = \frac{2+1}{2 k\Omega} = \frac{3}{2 k\Omega}$

$R_p = \frac{2}{3} k\Omega \simeq 0.667 k\Omega$

handy parallel trick: if $R_1 = R_2 = R$, $R_p = \frac{1}{2} R$.

$R_s >$ (all the R's)

$R_p <$ (all the R's)



current <u>through</u>

voltage <u>across</u>

Power considerations

Battery: $P_\mathcal{E} = \mathcal{E} I = (10V)(15mA) = 150mW$

R_1: $P_1 = I_1(\Delta V_1) = I_1^2 R_1 = (10mA)^2(1k\Omega) = 100mW$

R_2: $P_2 = I_2(\Delta V_2) = I_2^2 R_2 = (5mA)^2(2k\Omega) = 50mW$

$P_1 + P_2 = 100mW + 50mW = 150mW$ ✓

Next: combinations. Sometimes can replace combinations by simpler series or parallel equivalents.