Ch. 30 Part 1
Nuclear Physics

In this final chapter, we will explore the nucleus. Nuclear processes have a wide range of applications; we will only sample a few in this chapter. Many of these ideas will build upon existing work, but we will encounter new physics as well.

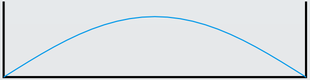
Before we dive into the chapter, there are two preliminary issues to explore:

1. What sorts of energies are involved in nuclear processes? We saw that atomic and molecular processes typically involved energies on the scale of electron Volts (eV).
2. Why doesn't the nucleus fly apart?

Nuclear Energy Scales

Consider the simplest cartoon model of a nucleus: Suppose a proton is confined to a one-dimensional box of length $L = 1.00 \times 10^{-14}\,\text{m} = 1.0 \times 10^{-5}\,\text{nm}$. (Recall that particles have wavelike behavior, and just like the vibrating string clamped at both ends, only certain wavelengths will be observed.) What energy would be required to raise it from the $n = 1$ state to the $n = 2$ state?

Consider the $n = 1$ state. Find the wavelength, momentum, and kinetic energy.



$L = 1\left(\frac{\lambda_1}{2}\right)$

$$L = 1\left(\frac{\lambda_1}{2}\right) \implies \lambda_1 = 2L = 2 \times 10^{-5}\,\text{nm}$$

$$p_1 = \frac{h}{\lambda_1} \qquad E_1 = \frac{p_1^2}{2m}$$

Nuclear Energy Scales

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$$E_1 = \frac{p_1^2}{2m} = \frac{(h/\lambda_1)^2}{2m} = \frac{h^2}{2m\lambda_1^2}$$

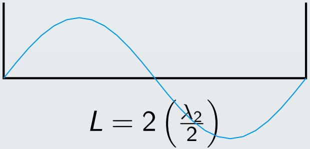
$$E_1 = \frac{(6.63 \times 10^{-34}\,\text{J s})^2}{2 \times (1.67 \times 10^{-27}\,\text{kg}) \times (2.00 \times 10^{-14}\,\text{m})^2}$$

$$E_1 = 3.28 \times 10^{-13}\,\text{J}$$

$$E_1 = 3.28 \times 10^{-13}\,\text{J} \times \frac{1\,\text{eV}}{1.602 \times 10^{-19}\,\text{J}} \times \frac{1\,\text{MeV}}{1 \times 10^6\,\text{eV}} = 2.05\,\text{MeV}$$

Nuclear Energy Scales

Next, consider the $n = 2$ state. Find the wavelength, momentum, and kinetic energy.



$$L = 2\left(\frac{\lambda_2}{2}\right) \implies \lambda_2 = L = 1 \times 10^{-5}\,\text{nm}$$

$$p_2 = \frac{h}{\lambda_2}$$

$$E_2 = \frac{p_2^2}{2m} = \frac{(h/\lambda_2)^2}{2m} = \frac{h^2}{2m\lambda_2^2}$$

$$E_2 = \frac{(6.63 \times 10^{-34}\,\text{J s})^2}{2 \times (1.67 \times 10^{-27}\,\text{kg}) \times (1.00 \times 10^{-14}\,\text{m})^2}$$

$$E_2 = 1.31 \times 10^{-12}\,\text{J} = 8.19\,\text{MeV}$$

Nuclear Energy Scales

Finally, consider an energy transition between those two states:

$$\Delta E = E_2 - E_1 = 8.19\,\mathrm{MeV} - 2.05\,\mathrm{MeV} = 6.14\,\mathrm{MeV}$$

This corresponds to a photon of wavelength

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\,\mathrm{eV\,nm}}{6.14\,\mathrm{MeV}}$$
$$\lambda = 0.000\,202\,\mathrm{nm}$$

This is a high-energy photon, in the $\gamma-$ray range.

Obviously, a real nucleus is not a one-dimensional box. Nevertheless, this quick calculation illustrates that the very small distances mean that large energies, on the order of millions of electron volts, will be encountered.

The Coulomb Force and Nuclear Stability

All the protons of an atom are contained in the nucleus. But protons are positively charged, which means that they tend to repel each other. Why doesn't the nucleus fly apart? There must be another force—we call it the *strong nuclear force*—holding the nucleus together.

What are the typical energy scales? Again consider a simple picture. Look at the helium nucleus (two protons, two neutrons). The protons are approximately 2.00×10^{-15} m apart. What is the electrostatic potential energy?

$$U = \frac{ke^2}{r} = \frac{(8.99 \times 10^9\,\mathrm{Nm^2/C^2}) \times (1.60 \times 10^{-19}\,\mathrm{J})^2}{2.00 \times 10^{-15}\,\mathrm{m}}$$

$$U = 1.15 \times 10^{-13}\,\mathrm{J} = 0.720\,\mathrm{MeV}$$

Again, we encounter energies on the order of millions of electron volts.

The nucleus is composed of Z protons and N neutrons. Common symbols:

Number of protons	Z
Number of neutrons	N
Atomic "mass" number	$A = Z + N$

In a neutral atom, the number of electrons is also Z. Nearly all the mass of the atom is contained in the nucleus:

Particle	**Mass**
Electron	$9.109\,383\,701\,5 \times 10^{-31}$ kg
Proton	$1.672\,621\,923\,69 \times 10^{-27}$ kg
Neutron	$1.674\,927\,498\,04 \times 10^{-27}$ kg

30.1 Nuclear Structure

It is useful to express masses in terms of the "atomic mass unit" u, which is $\frac{1}{12}$ the mass of the $^{12}\mathrm{C}$ nucleus:

	Mass
u	$1.660\,539\,066\,6 \times 10^{-27}$ kg
Proton	$1.007\,276\,466\,621$ u
Neutron	$1.008\,664\,915\,95$ u

The neutron is slightly more massive than the proton. Many processes will depend sensitively on those small differences, so we need to be careful not to round off too much. See Appendix C for relevant data for this chapter.

Isotopes

Some atoms can have different numbers of neutrons, but the same number of protons. Since the chemistry is primarily set by the number of electrons (or protons), these atoms all have the same chemical name. For example, all Carbon has 6 protons, but variants with 6, 7, or 8 neutrons can be found. These variants are called "isotopes."

An element S with Z protons and N neutrons (and hence atomic mass number $A = N + Z$) is written ${}^{A}_{Z}\mathrm{S}$. The Z is technically redundant, since the atomic symbol S includes the same information, but we often list the Z value explicitly for convenience. For example, most carbon atoms have 6 protons and 6 neutrons, so that could be written as ${}^{12}\mathrm{C}$ or ${}^{12}_{6}\mathrm{C}$.

Isotopes

Different isotopes have the same number of electrons, but have slightly different masses. This sometimes matters in processes or reactions where the mass plays a role.

Some isotopes are typically more commonly observed than others. For example, nearly all carbon we normally encounter is $^{12}\mathrm{C}$.

Isotope	**Abundance**
$^{12}_{6}\mathrm{C}$	0.989
$^{13}_{6}\mathrm{C}$	0.011
$^{14}_{6}\mathrm{C}$	1.3×10^{-12}

Isotopes

Ordinary chlorine, however, is typically a mixture of two isotopes

Isotope	**Abundance**
$^{35}_{17}\mathrm{Cl}$	0.7577
$^{37}_{17}\mathrm{Cl}$	0.2423

To help explore other nuclei, it is useful to examine what holds a nucleus together in the first place.

Consider the simplest compound nucleus—deuterium. It consists of one neutron and one proton. The symbol is ${}^{2}_{1}\mathrm{H}$, or just ${}^{2}\mathrm{H}$. Look at the masses. Express all masses in terms of the atomic mass $u = 1.660\,539\,066\,6 \times 10^{-27}$ kg.

proton	1.007 276 466 621 u
neutron	1.008 664 915 95 u
sum	2.015 941 382 571 u

30.2 Nuclear Stability

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proton	1.007 276 466 621 u
neutron	1.008 664 915 95 u
sum	2.015 941 382 571 u
deuteron	2.013 553 212 745 u
difference	0.002 388 169 826 u

Note that the deuteron mass is actually less than the sum of the proton and neutron masses. What does that mean?

Mass/Energy

Key to understanding nuclear reactions is Einstein's famous formula relating mass and energy

$$E = mc^2$$

where c is the speed of light. In this context, if we wanted to split a deuteron apart and produce a separate neutron and proton, we would have to add in the missing mass of 0.002 39 u. Or, using Einstein's formula, we would add in energy

$$\Delta E = (0.002\,39\,\mathrm{u}) \times (3.00 \times 10^8\,\mathrm{m/s})^2 = 3.56 \times 10^{-13}\,\mathrm{J}$$

A handy conversion factor is

$$1\,\mathrm{u}c^2 = (1.66 \times 10^{-27}\,\mathrm{kg}) \times (3.00 \times 10^8\,\mathrm{m/s})^2 = 1.492 \times 10^{-10}\,\mathrm{J}$$
$$1\,\mathrm{u}c^2 = (1.492 \times 10^{-10}\,\mathrm{J}) \times \left(\frac{1\,\mathrm{eV}}{1.60 \times 10^{-19}\,\mathrm{J}}\right) \times \left(\frac{1\,\mathrm{MeV}}{1 \times 10^6\,\mathrm{eV}}\right)$$
$$1\,\mathrm{u}c^2 = \boxed{931.5\,\mathrm{MeV}}$$

Mass/Energy

Thus in order to make up for the missing mass, we have to add

$$\Delta E = 0.002\,39\,\mathrm{u} \times c^2 = (0.002\,39) \times (931.5\,\mathrm{MeV}) = 2.225\,\mathrm{MeV}$$

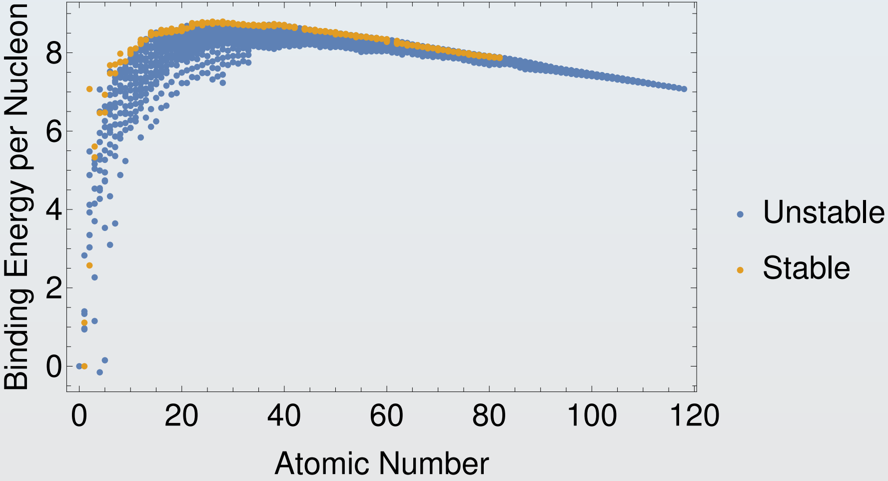
This is known as the "binding energy." Think of it analogous to the ionization energy for hydrogen. It tells how much energy you have to add to pull the deuteron apart. Equivalently, it tells how tightly bound the nucleus is. This binding force is known as the "strong nuclear force."

Binding Energy

More generally, the binding energy for a nucleus X is the difference between the mass of (Z hydrogen atoms + N neutrons) and the mass of the isotope ${}^{A}_{Z}\mathrm{X}$, all multiplied by c^2. Such changes in mass and energy $\Delta E = \Delta mc^2$ play a central role in nuclear reactions.

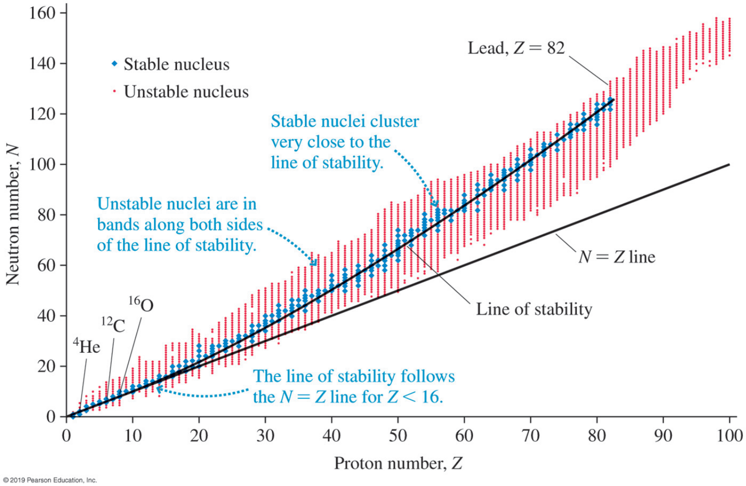
Some nuclei are more tightly bound than others. The less-tightly bound nuclei tend to be unstable.

Binding Energy



Binding energies (MeV) for stable (orange) and unstable (blue) nuclei.

30.2 Nuclear Stability



As the number of protons increases, more neutrons are needed to “glue” the nucleus together.

You may skip this detailed discussion of the filling of nuclear energy levels.

What's Next?

- Nuclear decay processes
- Half life
- Applications