

Experiment 4

The Lifetime of the Muon

Safety

There are no unusual safety hazards in this experiment.

Additional documentation for this experiment:

- Manual for the Muon apparatus (available on Moodle)
- You can find a wealth of data on elementary particles at the web site for the Particle Data Group <http://pdg.lbl.gov/>. In particular, a summary of muon properties is at <http://pdg.lbl.gov/2015/tables/rpp2015-sum-leptons.pdf>.

4.1 Introduction

Muons are unstable elementary particles with a charge equal to that of an electron, a mass roughly 200 times that of an electron, and a mean lifetime of about 2.2 μs . In this experiment, you will measure the decay of muons that are created by cosmic rays, and see how time dilation affects the measured results.

The muons we detect in this experiment arise from cosmic rays. High-energy protons from outer space can collide with the nuclei of air molecules in the upper atmosphere and create an array of particles. One of most common products is charged pions ($\pi^\pm$), as illustrated in Fig. 4.1. Those pions then very quickly decay (with a lifetime of roughly 26 ns) into muons and neutrinos

$$\begin{aligned}\pi^+ &\rightarrow \mu^+ + \nu_\mu \\ \pi^- &\rightarrow \mu^- + \bar{\nu}_\mu ,\end{aligned}$$

where ν_μ and $\bar{\nu}_\mu$ are the neutrino and anti-neutrino of the muon type. You will learn more about neutrinos later in this course, but they interact extremely weakly with matter and are not detected in this experiment.¹ The muons ($\mu^\pm$) from this reaction are the ones we wish to detect.

¹We will ignore neutrinos throughout this experiment.

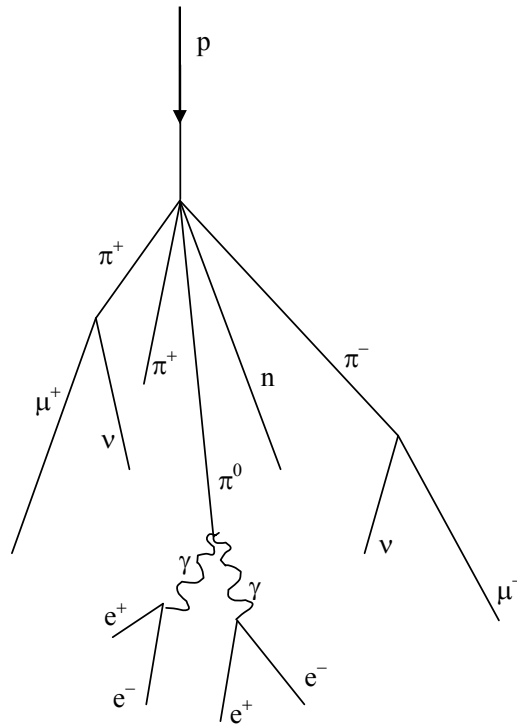


Figure 4.1: Cosmic ray cascade induced by a cosmic ray proton striking an air molecule. (Reproduced from the TeachSpin manual “Muon Physics” by T.E. Coan and J. Ye.)

These muons are highly relativistic; the ones we typically detect have a Lorentz factor of about $\gamma = 11$.

Some of the muons scatter off atmospheric molecules and do not reach the surface, but many others do reach Earth’s surface. In our apparatus we typically see about 4 or 5 muons per second.

Muons have a mean lifetime of $2.197 \mu\text{s}$, and decay via the reaction

$$\begin{aligned}\mu^- &\rightarrow e^- + \nu_\mu + \bar{\nu}_e \\ \mu^+ &\rightarrow e^+ + \bar{\nu}_\mu + \nu_e ,\end{aligned}$$

where e and e^+ are an electron and positron, and ν_e and $\bar{\nu}_e$ are the neutrino and anti-neutrino of the electron type.²

In this experiment, we will detect muons that enter the detector, give up their energy and stop, and then decay. We will typically observe about 1 decay/min.

²There are subtle difference between the two decay rates in the presence of matter, but they are not significant for our experiment.

4.2 Theory

4.2.1 Particle Decay and Mean Lifetime

Muons decay following a probabilistic mathematical description. This is the same type of process as as involved in radioactive decay. If you start with an initial number N_0 of muons, the number ΔN that will likely decay in an short time interval Δt is proportional to the number you start with and the time interval, *i.e.*

$$\frac{\Delta N}{N_0} = -\frac{\Delta t}{\tau}$$

where the minus sign simply means that the number decreases, and the constant τ is known as the “lifetime.” It has units of seconds, and depends on the particular particle. Lifetimes can vary extremely widely, from 1×10^{-25} s for the heaviest quarks to almost 15 minutes for free neutrons, to billions of years for some unstable radioactive nuclei.

This relation holds true for any given start time. You don’t have to measure the particles when they are created. Whatever number N_0 you have at any given time, the same fraction $\frac{\Delta N}{N_0} = -\frac{\Delta t}{\tau}$ is expected to decay in the next time interval δt . This leads to exponential decay

$$N(t) = N_0 e^{-t/\tau}. \quad (4.1)$$

A related number is the half-life $t_{1/2}$, defined as the time needed for half the particles to decay. Using Eq. 4.1, $t_{1/2}$ can be expressed in terms of τ :

$$\begin{aligned} \frac{1}{2} N_0 &= N_0 e^{-t_{1/2}/\tau} \\ \frac{1}{2} &= e^{-t_{1/2}/\tau} \\ \ln\left(\frac{1}{2}\right) &= -\frac{t_{1/2}}{\tau} \\ \ln(1) - \ln(2) &= -\frac{t_{1/2}}{\tau} \\ -\ln(2) &= -\frac{t_{1/2}}{\tau} \\ t_{1/2} &= \ln(2) \tau \approx 0.693 \tau \end{aligned} \quad (4.2)$$

4.3 Experiment

The first experimental challenge is to detect the passage of muons through the scintillator crystal. The passage of a charged particle through the scintillator material deposits energy, some of which is converted to photons. The photomultiplier tube detects those photons and puts out a brief voltage pulse that is amplified by the Muon Physics electronics box.

4.3.1 Pulse Detection and Muon Counting

The apparatus should already be set up and functioning. These settings are just for reference.

Connect the “DC Power Output” on the back of the electronics box to the “DC Power Input” jack on the top of the scintillator tube. This supplies the high voltage necessary for the photomultiplier. The PMT voltage is set with the “HV Adj.” knob on the scintillator. You can monitor the photomultiplier voltage with voltage probes at the “HV 1:100 Monitoring” jacks. Those make available a voltage equal to 1/100 of the PMT voltage. The photomultiplier tube high voltage should be set to about 1100 V.

Connect the “PMT Output” of the scintillator tube to the “PMT Input” jack on the electronics box.

Connect the USB output of the electronics box to the USB input of the computer.

Turn on the electronics box (the switch is on the back).

Muons entering the detector will cause pulses around 10 mV in amplitude, while less-energetic particles (such as those due to natural background radioactivity) will typically generate much-less energetic pulses. The “discriminator” setting on the electronics box should be set to around 7.5 mV to reject those smaller pulses with energies less than 7.5 mV, while still counting the larger pulses, which are typically due to muons. You can monitor the discriminator voltage with the voltmeter probes at the terminals marked “Threshold Voltage”. Those terminals put out a voltage equal to $V_{\text{thresh}}/50$ mV.

Once everything is set, the box can emit an audible beep every time a muon is detected; typically we see about 4.5 muons/s.

Similarly, if a muon comes to rest and decays inside the scintillator, it will also emit a strong enough signal to be detected.

Counting Muon Decays

Now the system is set up so that every muon produces a pulse at the discriminator, while most noisy fluctuations are ignored. Similarly, when a muon decays inside the detector, another pulse will be recorded.

The next challenge is to count muon *decays*. Specifically, suppose you detect a muon pulse. What happens next? When it detects a pulse, the software then waits 20 μs (about 10τ) to look for a second pulse.

Normally, you see a muon event about 5 times a second, or with an average delay of roughly $(\frac{1}{5})\text{s} = 200\text{ ms} = 200\,000\text{ }\mu\text{s}$ between events. Of course muons arrive randomly, so in that window of 20 μs you would occasionally expect to see a second muon, but you’d expect to have to wait a long time, on average.

Instead, what you should observe after about 5 minutes, is that there are a number of “successor” events. There are two possible explanations for the successor events:

- A) a first muon passed through, followed by a second muon passing through within 20 μs ,
or
- B) a first muon entered and stopping, followed by that muon decaying within 20 μs .

Pre-lab Question 1: If the average time between pulses is 200 000 μs , what is the probability that two pulses occur within 20 μs (*i.e.* option A)?

The software will record the time intervals between those successor events and generate a histogram, giving the number of events within a certain time interval. After a while, the distribution should look something like Fig. 4.2.

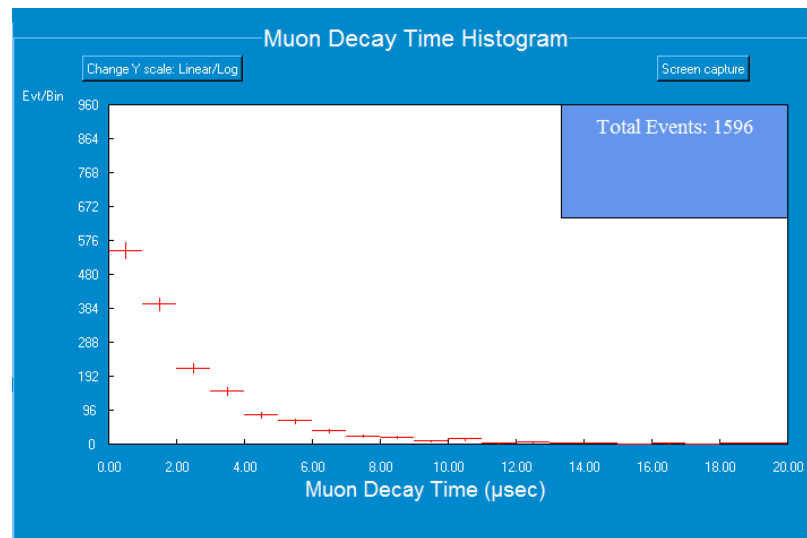


Figure 4.2: Distribution of successor events.

Pre-lab Question 2: Consider the *distribution* of events in Fig. 4.2. If the pulses were due to different muons (option A), what sort of distribution would you expect? What do you observe? What do you conclude?

4.3.2 Time Dilation

If we are indeed seeing muons decay with a mean lifetime of $2.2\ \mu\text{s}$, then we have to confront a new question: How do the muons survive to make it to the detector? Specifically, do the following calculations:

Pre-lab Question 3:

- Assume the muons are created in the upper atmosphere, at a height of $h = 15\ \text{km}$ with a Lorentz factor of $\gamma = 11$. What is their speed (in meters/second)?
- Assuming no relativity, how long would those muons take to travel to our detector at sea level?
- The lifetime of a muon is about $\tau = 2.2\ \mu\text{s}$. Use Eq. 4.1 to predict the fraction of muons that would survive to reach our detector.
- Now incorporate time dilation. The time in Eq. 4.1 should be the proper time, as experienced by the muon. For a γ factor of 11, what fraction of muons will survive to reach our detector?
- Based on your calculations, is time dilation an important factor in this experiment?

4.3.3 Data Acquisition

The system should already be set up, and data taken over the past week will be available on Moodle. These instructions are here for reference.

PMT and Discriminator Voltages

Record the PMT and discriminator voltages you are using. From here on, leave them set.

Software

Double-click on the **muon** icon on the desktop. On the start-up screen, select **Configure** and then select Port **COM 4**. Set the histogram scale to $20\mu s$ and 20 bins. Press **Save**. You then start acquiring data with the **Start** button.

Each muon event followed by a second event within $20\mu s$ will be counted as a “decay.”

On average, the system detects about one muon decay per minute. In order to get good statistics, you need a long run. While you can get a reasonable estimate of τ in about an hour, longer runs give much better statistics.

You need to ‘Quit’ in order to save your work. The program will store the data in the **Muon_data** folder in a filename with a date/timestamp of the form **YY-MM-DD-hh-mm.data**. It will also append your data to the pre-existing **muon.data** file. The format for that data file is given in the TeachSpin manual available on the course Moodle site.

4.3.4 Long Data Acquisition

Data from runs over periods of at least one full day will be posted to Moodle. You will each be assigned a single run to analyze.

Data Files and File Formats

For convenience, the decay events for each run have been extracted into two files. The first file, with a name such as **2026-09-25.csv**, is a list of the decays. The file has two columns: decay time, and elapsed time, as illustrated in Table 4.1.

Decay_Time,	Elapsed_Time
1360,	0
2480,	148
4500,	281
1380,	327

Table 4.1: Format of the data file for muon decay events. The first column is the time (in nanoseconds) for the muon to decay. The second column is the elapsed time (in seconds) since the first detected decay.

The second file, with a name such as **2026-09-25-hist.csv**, is a histogram table, showing how many decays were detected within each time range (or “Bin”) as illustrated in Table 4.2.

Bin,	Count
0,	1528
1000,	1115
2000,	704
3000,	397

Table 4.2: Format of the data file for the muon decay histogram. Each decay has been placed in a histogram bin. The first column is the time (in nanoseconds) for the start of the bin, and the second column is the number of decays within that bin. For this example data, there were 1,528 decays with times between 0 and 1000 ns, 1,115 decays with times between 1000 ns and 2000 ns, and so on.

4.4 Analysis

The events you have observed include genuine muon decays, as well as a background of events where two independent muons happened to pass through the detector within $20\mu\text{s}$ of each other. Consequently, the number of decays ought to follow an exponential distribution as in Eq. 4.1, along with a steady background B :

$$N(t) = Ae^{-t/\tau} + B , \quad (4.3)$$

where τ is the mean lifetime. You will use two different approaches to estimate τ , finding the half-life $t_{1/2}$, and fitting the histogram to Eq. 4.3.

4.4.1 Half Life

To estimate the half-life, import the decay events file into a spreadsheet. The file is a “Comma Separated Values” (CSV) file. The spreadsheet should display the data as two columns, as in Table 4.1. Ask your instructor if you would like assistance importing the data.

Record the following observations and calculations in your report:

1. First, look at the elapsed time column: What is the total elapsed time (in seconds) for your data? Record that value. You will need it later in §4.4.2 to estimate the number of background events.

From here on, you will not need that second column. You may delete it from your spreadsheet.

2. Sort the first column of your spreadsheet (the decay times) in increasing order.
3. How many total events did you have? (That is, how many data rows are there in your spreadsheet?)
4. Over what time interval did half of your total muons decay? That is, suppose you had 3,000 decays. What decay time corresponded to event 1,500? (This is simply the time in cell A1500. Round off the event number to the nearest integer) Call that t_{1500} . This is your first estimate of the half-life. Be sure you are reading from the first column of your spreadsheet.

5. Now, starting at the time you found in the previous step, over what time interval did half of the remaining muons decay? Again, assuming you had 3,000 decays, you are now looking at the decay time from decay 1,500 to decay 2,250 (half-way between 1,500 and 3,000, or from $1/2$ to $3/4$ of the way through the list). What was *that* time interval, *i.e.* what was $t_{2250} - t_{1500}$? Again, be sure you are reading from the first column of your spreadsheet. This is your second estimate of the half-life.
6. Finally, starting at the time you found in the previous step, over what time interval did half of the remaining muons decay? Again, assuming you had 3,000 decays, you are now looking at the decay time from decay 2,250 to decay 2,625 (half-way between 2,250 and 3,000, or from $3/4$ to $7/8$ of the way through the list). This is your third estimate of the half-life. Since you are now looking at far fewer events, this estimate is likely the least reliable.
7. What is your average value for $t_{1/2}$? What is your uncertainty? (See Appendix A of the lab manual. Remember the uncertainty is not just the standard deviation of your three values; the uncertainty is the standard deviation divided by $\sqrt{3}$.)
8. Recall from Eq. 4.2 that

$$\tau = \frac{t_{1/2}}{\ln(2)}.$$

Based on your half-life measurements, what is your value for τ ? Include the uncertainty. (Note that since you divide $t_{1/2}$ by $\ln(2)$, you must also divide the uncertainty by $\ln(2)$.)

9. Compare your measured value of τ to the expected value of 2197 ns. Be sure to account for the uncertainty when making that comparison.

4.4.2 Curve Fitting

Next, you will estimate τ by fitting your histogram of decay times to Eq. 4.3.

1. Launch **Vernier Graphical Analysis**. When the program starts, select “Manual Entry.”
2. Import the histogram file (as in Table 4.2): Click on the **File** menu from the upper left corner (the icon next to the word “Untitled”) and select “Open.” At the bottom of the file dialog box, click on “Show Options”. The “Format” menu should appear; select “Comma Separated Values” and then select your histogram file.
3. Your graph should now show a plot of “Count” *vs.* “Bin”. Typically for a data plot, you should show the individual data points, not a line. Click on the “Graph Options” icon (the graph at the lower left of the window). Scroll down to the bottom of the menu that appears and select “Edit Graph Options.” Under “Appearance,” check the box for “Points” and un-check the box for “Lines.” Click anywhere in the graph to return to the graph. If an extra vertical line appears on the graph, you can get rid of it by clicking on the small X near the top of the vertical line.
4. Fit Eq. 4.3 to your data. Click on “Graph Options” and select “Apply Curve Fit.” In the drop-down menu, select “Natural Exponent.” Carefully compare the fit equation to the desired form Eq. 4.3. The letter “x” plays the role of time t , but note that the coefficient “c” is in the wrong place. Click on the “Create Custom Fit” option and

edit the “Fit Expression” so that it matches the desired form. Give your function a useful name, such as “Particle Decay.” A solid line for the fit will appear on the graph, along with a box showing the fit parameters. Click on the gear icon in that fit box, select the box for “Show Uncertainty of Fit Parameters,” and click “OK.”

5. Insert your graph into your report, either by taking a screenshot or by using the “File” menu to Export an image (either .PDF or .PNG) of your graph.
6. Compare the value for τ from your fit to the expected value. As always, include the uncertainty in this analysis.
7. Estimate the expected background (B in Eq. 4.3) in the following way:
 - (a) On average, there are 5 muon events detected per second. Over the total time of your data set, how many muon events would be detected?
 - (b) In Pre-lab question 1, you estimated the probability that a second muon happened to pass through the scintillator within $20\mu\text{s}$ of the first muon. The software will include this coincidental event in the histogram of events. How many total such coincidental events do you expect in your data set?
 - (c) Those events are presumably spread out evenly across your histogram bins. So if you divide the number of coincidental events by the number of histogram bins, you should get an estimate of the background B . What value do you estimate for B ?
8. Compare your estimate for B with the value you found from the curve fit. Again, include the uncertainty in this analysis.