13.64 •• CP **Binary Star—Different Masses.** Two stars, with masses M_1 and M_2, are in circular orbits around their center of mass. The star with mass M_1 has an orbit of radius R_1; the star with mass M_2 has an orbit of radius R_2. (a) Show that the ratio of the orbital radii of the two stars equals the reciprocal of the ratio of their masses—that is, $R_1/R_2 = M_2/M_1$. (b) Explain why the two stars have the same orbital period, and show that the period T is given by $T = 2\pi(R_1 + R_2)^{3/2}/\sqrt{G(M_1 + M_2)}$. (c) The two stars in a certain binary star system move in circular orbits. The first star, Alpha, has an orbital speed of 36.0 km/s. The second star, Beta, has an orbital speed of 12.0 km/s. The orbital period is 137 d. What are the masses of each of the two stars? (d) One of the best candidates for a black hole is found in the binary system called A0620-0090. The two objects in the binary system are an orange star, V616 Monocerotis, and a compact object believed to be a black hole (see Fig. 13.28). The orbital period of A0620-0090 is 7.75 hours, the mass of V616 Monocerotis is estimated to be 0.67 times the mass of the sun, and the mass of the black hole is estimated to be 3.8 times the mass of the sun. Assuming that the orbits are circular, find the radius of each object's orbit and the orbital speed of each object. Compare these answers to the orbital radius and orbital speed of the earth in its orbit around the sun.

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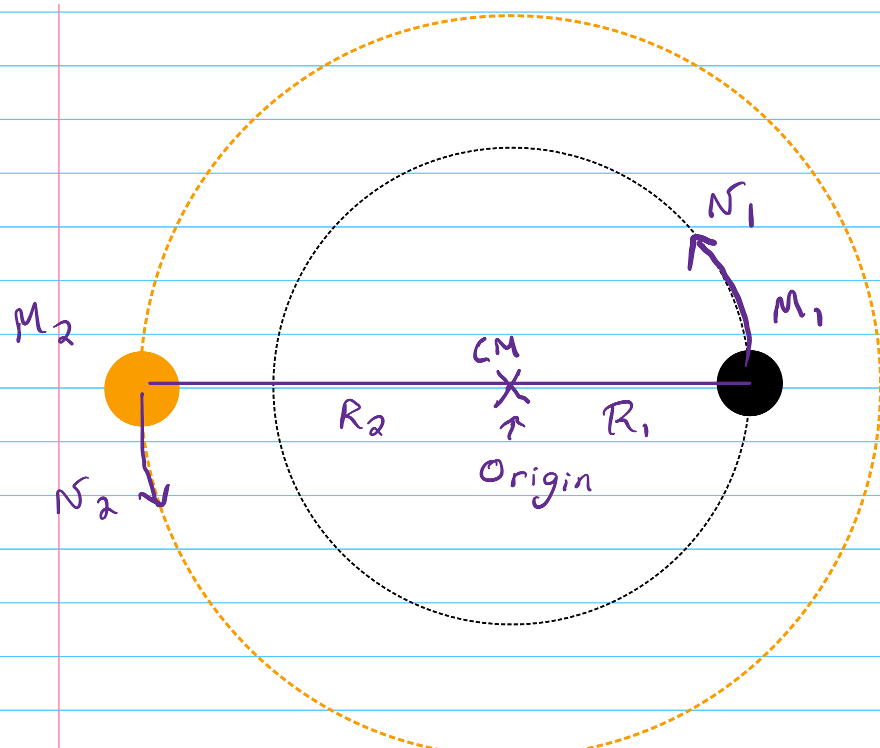

Suppose the origin is at the center of mass.

$$X_{cm} = \frac{M_1 X_1 + M_2 X_2}{M_1 + M_2}$$

(a) If $X_{cm} = 0$, then $M_1 X_1 + M_2 X_2 = 0$

$$\frac{M_1}{M_2} = \frac{-X_2}{X_1}$$

13.64 (continued)

Let $R_2 = |x_2|$ = the distance of M_2 from the origin.

Then $\boxed{\dfrac{M_1}{M_2} = \dfrac{R_2}{R_1}}$

(b) Look at star 1:

$$F = M_1 a_1$$

$$\frac{GM_1M_2}{(R_1+R_2)^2} = M_1\frac{v_1^2}{R_1}$$

travels in a circle of radius R_1 about the center.

The stars are separated by a distance (R_1+R_2).

Then, use $v_1 = \dfrac{2\pi R_1}{T_1}$, so

$$\frac{GM_1M_2}{(R_1+R_2)^2} = \frac{M_1}{R_1}\left(\frac{2\pi R_1}{T_1}\right)^2 = \frac{4\pi^2 M_1R_1}{T_1^2} \quad (1)$$

Next, look at star 2

$$\frac{GM_1M_2}{(R_1+R_2)^2} = M_2\frac{v_2^2}{R_2} = \frac{M_2}{R_2}\left(\frac{2\pi R_2}{T_2}\right)^2$$

$$\frac{GM_1M_2}{(R_1+R_2)^2} = \frac{4\pi^2}{T_2^2}M_2R_2 \quad (2)$$

Since these two forces are equal,

$$\frac{4\pi^2}{T_1^2}M_1R_1 = \frac{4\pi^2}{T_2^2}M_2R_2$$

Then from part (a), $M_1R_1 = M_2R_2$, so $\boxed{T_1 = T_2}$

Non-obvious trick: combine Eqs. 1 + 2 (after cancelling M's)

$$1)\quad \frac{G\cancel{M_1}M_2}{(R_1+R_2)^2} = \frac{4\pi^2\cancel{M_1}R_1}{T^2}$$

$$2)\quad \frac{GM_1\cancel{M_2}}{(R_1+R_2)^2} = \frac{4\pi^2\cancel{M_2}R_2}{T^2}$$

. Now add:

$$\frac{GM_2}{(R_1+R_2)^2} + \frac{GM_1}{(R_1+R_2)^2} = \frac{4\pi^2 R_1}{T^2} + \frac{4\pi^2 R_2}{T^2}$$

$$\frac{G(M_1+M_2)}{(R_1+R_2)^3} = \frac{4\pi^2}{T^2}(R_1+R_2)$$

, so

$$\boxed{G(M_1+M_2)T^2 = 4\pi^2(R_1+R_2)^3}$$

13.64 (continued)

(c) Star 1: Alpha $\qquad$ Star 2: Beta

$v_1 = 36 \times 10^3 \, m/s$ $\qquad$ $v_2 = 12 \times 10^3 \, m/s$

$T = T_1 = T_2 = 137 d = 1.184 \times 10^7 s$

So we can find R_1 and R_2:

$$v_1 = \frac{2\pi R_1}{T_1}, \Rightarrow R_1 = \frac{v_1 T_1}{2\pi} = 6.782 \times 10^{10} m$$

$$v_2 = \frac{2\pi R_2}{T_2} \Rightarrow R_2 = \frac{v_2 T_2}{2\pi} = 2.261 \times 10^{10} m$$

Now go back to $F = ma$ for M_1:

$$\frac{G M_1 M_2}{(R_1 + R_2)^2} = M_1 \frac{v_1^2}{R_1}$$

The M_1's cancel, so

$$M_2 = \frac{(R_1 + R_2)^2 v_1^2}{G R_1}$$

$$M_2 = \frac{\left((6.782 + 2.261) \times 10^{10} m\right)^2 (36 \times 10^3 m/s)^2}{(6.67 \times 10^{-11} Nm^2/kg^2)(6.782 \times 10^{10} m)}$$

$$\boxed{M_2 = 2.34 \times 10^{30} kg}$$

. Then find M_1:

$$M_1 = \frac{R_2 M_2}{R_1} = \frac{(2.261 \times 10^{10} m)}{6.782 \times 10^{10} m} \cdot 2.34 \times 10^{30} kg$$

$$\boxed{M_1 = 7.81 \times 10^{29} kg}$$

(13.64 continued)

(d) $T = 7.75 \text{ hrs} = 27,900 \text{ s}$

$$M_1 = 0.67 M_{sun} = 1.34 \times 10^{30} \text{ kg}$$
$$M_2 = 3.8 M_{sun} = 7.60 \times 10^{30} \text{ kg}$$

Kepler's 3rd law from part (b) is useful here:

$$G(M_1 + M_2)T^2 = 4\pi^2 (R_1 + R_2)^3$$

$$R_1 + R_2 = \left(\frac{G(M_1 + M_2)T^2}{4\pi^2} \right)^{1/3}$$

$$R_1 + R_2 = \left(\frac{(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)((1.34 + 7.60) \times 10^{30} \text{ kg})(27900 \text{ s})^2}{4\pi^2} \right)^{1/3}$$

$$R_1 + R_2 = 2.274 \times 10^9 \text{ m}$$

we also know $M_1 R_1 = M_2 R_2$, so $R_2 = \frac{M_1 R_1}{M_2}$

$$R_1 + \frac{M_1}{M_2} R_1 = 2.274 \times 10^9 \text{ m}$$

$$R_1 = \frac{2.274 \times 10^9 \text{ m}}{1 + \frac{M_1}{M_2}} = \frac{2.274 \times 10^9 \text{ m}}{1 + \frac{0.67}{3.8}}$$

$$\boxed{R_1 = 1.93 \times 10^9 \text{ m} \qquad R_2 = \frac{M_1}{M_2} R_1 = 0.34 \times 10^9 \text{ m}}$$

$$\boxed{v_1 = \frac{2\pi R_1}{T} = 4.35 \times 10^5 \text{ m/s} \qquad v_2 = \frac{2\pi R_2}{T} = 7.66 \times 10^4 \text{ m/s}}$$