Problem 15.11

Given:

At positions x0 and x1, we are given simple harmonic motion with a period of T. However, the oscillation at x1 doesn't reach its peak until 0.035 s.

(a) and (b). Compute various constants related to the wave.

In[18]:=
```
x0 = 0; x1 = 0.09; T = 0.04; A = 4; (* x's in meters, y's in mm *)
```

In[19]:=
```
tmin = 0; tmax = 0.08;
```

In[20]:=
```
ω = 2 π / T
```

Out[20]=
```
157.08
```

In[21]:=
```
Clear[ϕ]
y0[t_] := A Sin[ω t]
y1[t_] := A Sin[ω t + ϕ]
```

Here is one way to find the phase ϕ such that the peak for wave 1 occurs at 0.035s, as in the figure. (This is not necessary; I am only doing it to reproduce the picture already in the text.) You could also do this analytically with the inverse sine function.

In[24]:=
```
FindRoot[y1[0.035] == A, {ϕ, 0}]
```

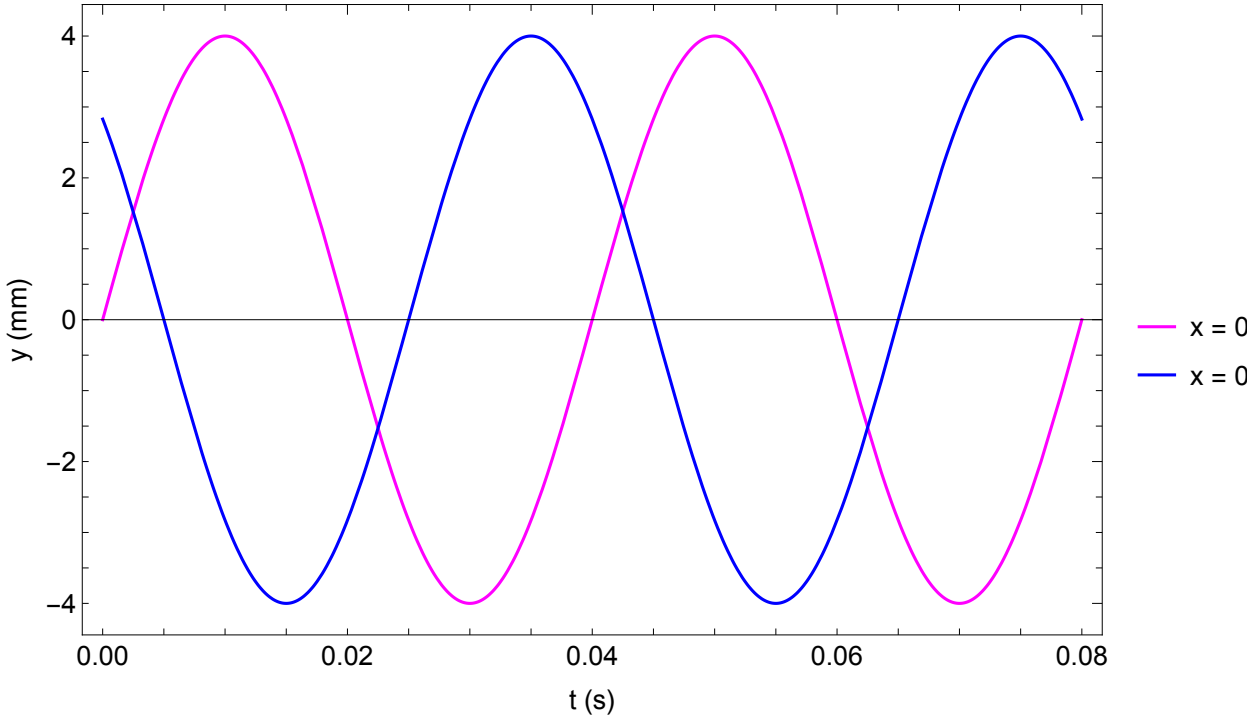
Out[24]=
```
{ϕ → 2.35619}
```

In[25]:=
```
ϕ = ϕ /. %
```

Out[25]=
```
2.35619
```

In[26]:=
```
Plot[{y0[t], y1[t]}, {t, tmin, tmax},
 PlotStyle → {Magenta, Blue}, LabelStyle → Larger,
 PlotLegends → {"x = 0", "x = 0.09"}, Frame → True,
 FrameLabel → {"t (s)", "y (mm)"}, ImageSize → Large]
```

Out[26]=



(c) Wave moves to the right.

If the wave moves to the right, and the wavelength is greater than x1-x0, then we can find the wave speed by asking how long it takes the peak to travel from x0 to x1. The peak at x0 (the magenta curve) occurred at t = 0.01. The peak at x=0.09 (the blue curve) occurred at t=0.035. Thus the speed is:

In[27]:=
```
v = (x1 - x0) / (0.035 - 0.01) (* m/s *)
```

Out[27]=
```
3.6
```

In[28]:=
```
λ = v T
```

Out[28]=
```
0.144
```

In[29]:=
```
xmax = 3 λ; (* Convenient size for plots *)
```

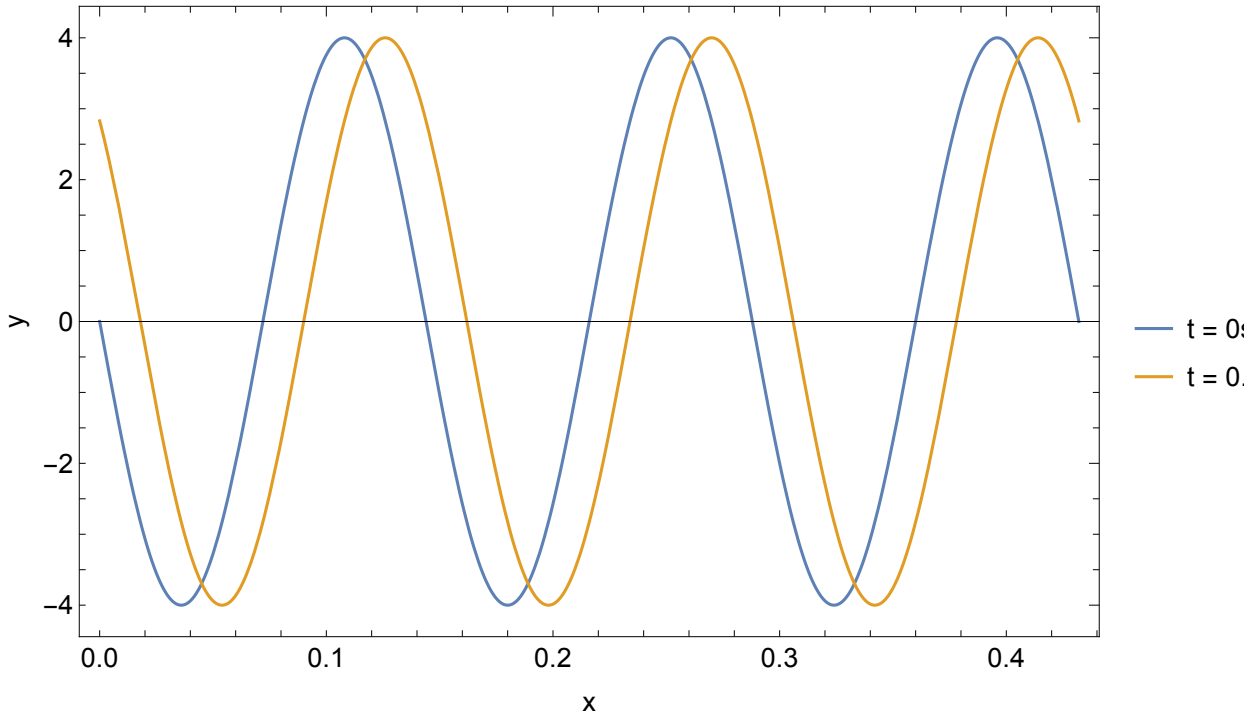
In[30]:=
```
y[x_, t_] := A Sin[2 π / λ x - 2 π / T t + π]
```

The '+ π' at the end is to ensure the magenta curve goes up rather than down.

Here are snapshots of the wave at times t = 0 and t = 0.005 seconds.

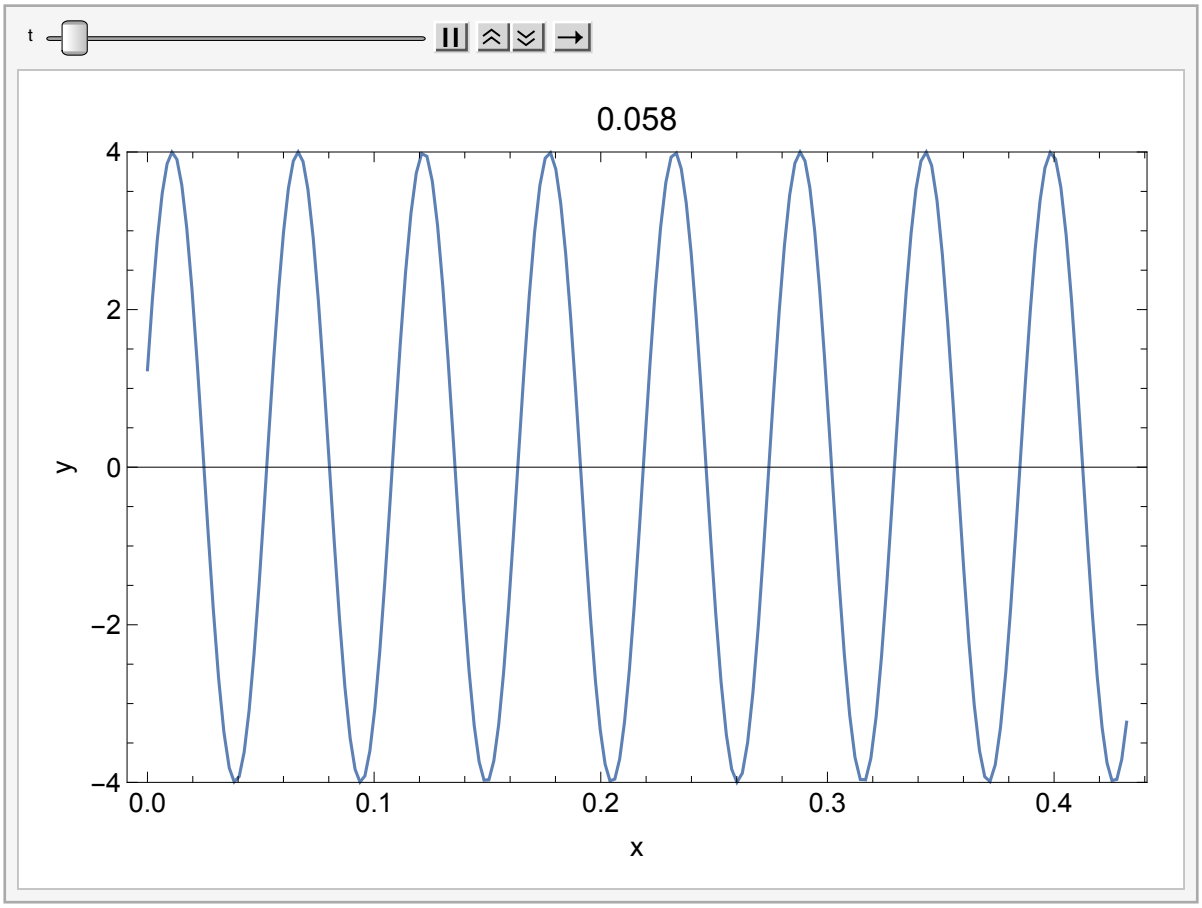
In[31]:=
```
Plot[{y[x, 0], y[x, 0.005]}, {x, 0, xmax},
 PlotLegends → {"t = 0s", "t = 0.005 s"}, LabelStyle → Larger,
 Frame → True, FrameLabel → {"x", "y"}, ImageSize → Large]
```

Out[31]=



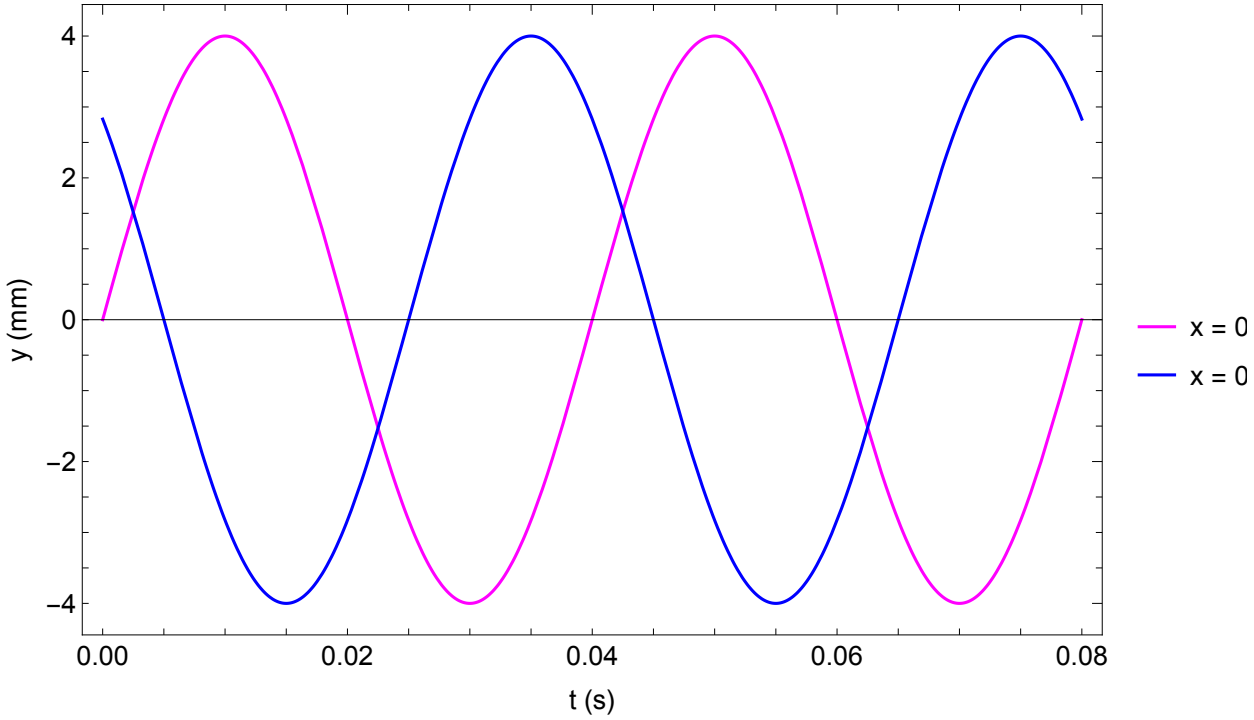
In[32]:=
```
Animate[Plot[y[x, t], {x, 0, xmax}, PlotRange → {-A, A},
  PlotLabel → t, LabelStyle → Larger, Frame → True,
  FrameLabel → {"x", "y"}, ImageSize → Large], {t, tmin, tmax, 0.001}]
```

Out[32]=



In[33]:=
```
Plot[{y0[t], y1[t]}, {t, tmin, tmax},
 PlotStyle → {Magenta, Blue}, LabelStyle → Larger,
 PlotLegends → {"x = 0", "x = 0.09"}, Frame → True,
 FrameLabel → {"t (s)", "y (mm)"}, ImageSize → Large]
```

Out[33]=

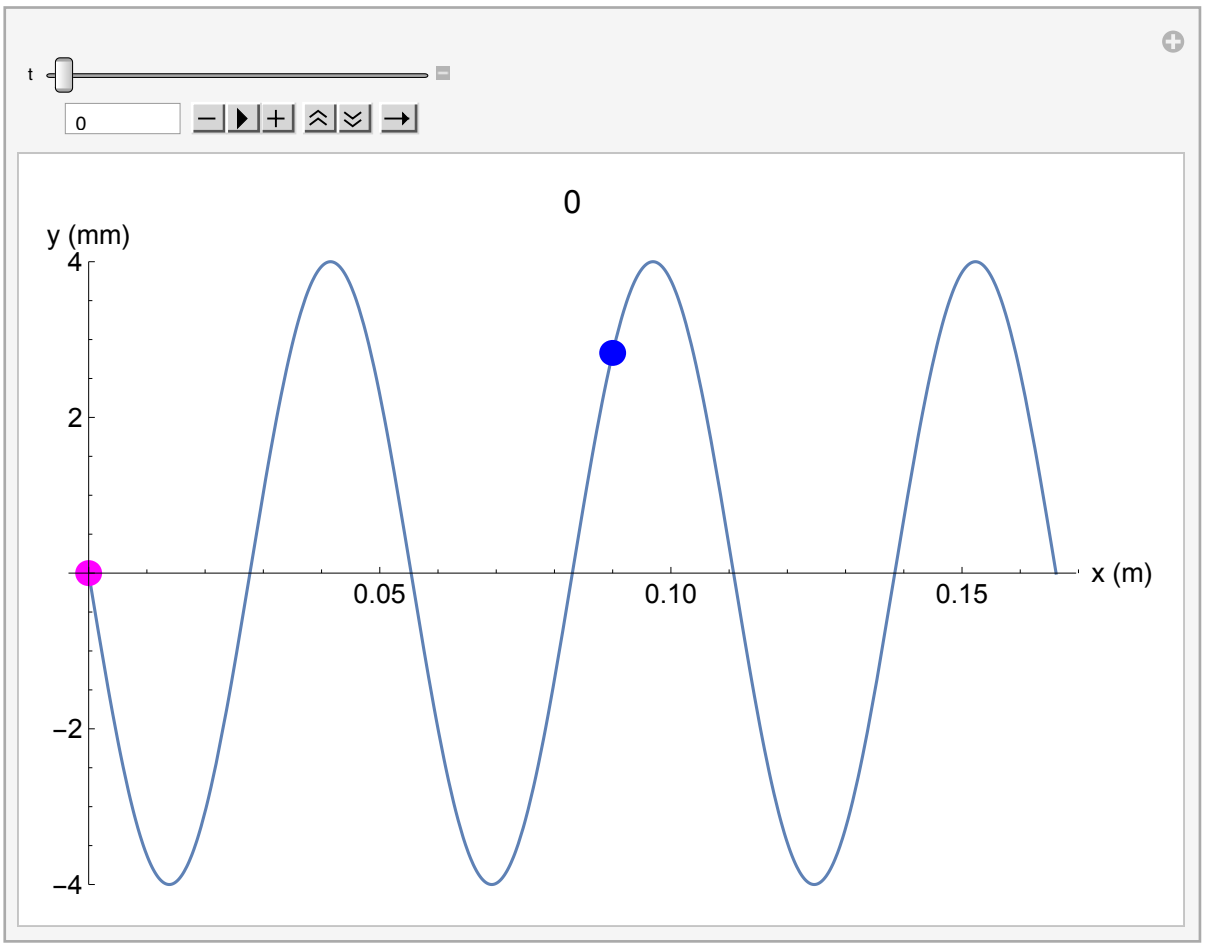


Look at x = x0 and x = x1

In[34]:=
```
dot[color_, x_, t_] := Graphics[{color, Disk[{x, y[x, t]}, {λ/24, A/24}]}]
```

In[35]:=
```
Manipulate[Show[{Plot[y[x, t], {x, 0, 3 λ}, PlotRange → {-A, A}, PlotLabel → t],
    dot[Magenta, x0, t],
    dot[Blue, x1, t]}, LabelStyle → Larger,
  AxesLabel → {"x (m)", "y (mm)"}, ImageSize → Large
 ],
 {t, tmin, tmax, 0.0001, Appearance → "Open",
  DisplayAllSteps → True, DefaultDuration → 30}]
```

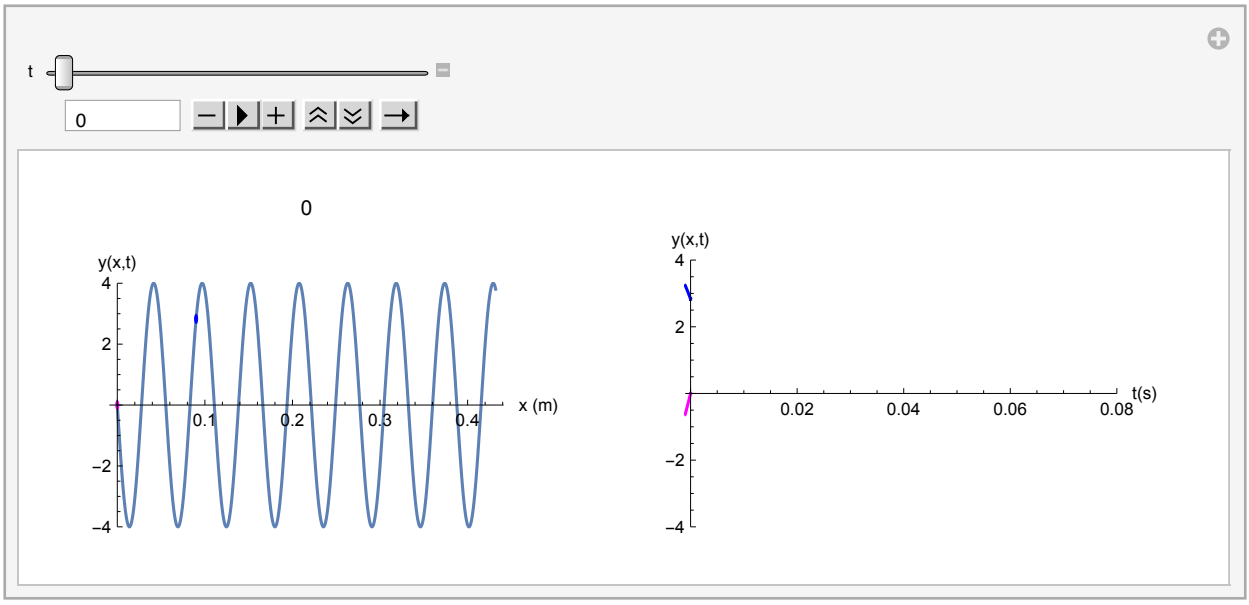
Out[35]=



We can show how the original graph resulted by tracing out the two dots.

In[36]:=
```
tc = tmax;
Manipulate[GraphicsRow[{
    Show[Plot[y[x, t], {x, 0, xmax}, PlotRange → {-A, A},
      PlotLabel → t, AxesLabel → {"x (m)", "y(x,t)"}],
     dot[Magenta, x0, t], dot[Blue, x1, t], ImageSize → 350],
    Show[
     Plot[{y[x0, tp], y[x1, tp]}, {tp, tmin - 0.001, t},
      PlotStyle → {Magenta, Blue}, PlotRange → {{tmin - 0.001, tmax}, {-A, A}},
      AxesLabel → {"t(s)", "y(x,t)"}, ImageSize → 350]
    ]}
 ], {t, tmin, tmax, 0.0002, Appearance → "Open",
  DisplayAllSteps → True, DefaultDuration → 30}]
```

Out[37]=



Shorter Wavelenghts?

If the wave moves to the right, and the wavelength is less than,x1-x0, then we may need to look two (or more) peaks away.

In[38]:=
```
ΔT = 0.025 + T (* Add in one period *)
```

Out[38]=
```
0.065
```

In[39]:=
```
v = (x1 - x0) / ΔT
```

Out[39]=
```
1.38462
```

In[40]:=
```
λ = v T
```

Out[40]=
```
0.0553846
```

In[41]:=
```
tc = tmax;
Manipulate[GraphicsRow[{
   Show[Plot[y[x, t], {x, 0, xmax}, PlotRange → {-A, A}, PlotLabel → t],
    dot[Magenta, x0, t], dot[Blue, x1, t], ImageSize → 350],
   Show[
    Plot[{y[x0, tp], y[x1, tp]}, {tp, tmin - 0.001, t},
     PlotStyle → {Magenta, Blue},
     PlotRange → {{tmin - 0.001, tmax}, {-A, A}}, ImageSize → 350]
   ]}
 ], {t, tmin, tmax, 0.001, Appearance → "Open"}]
```

Out[42]=