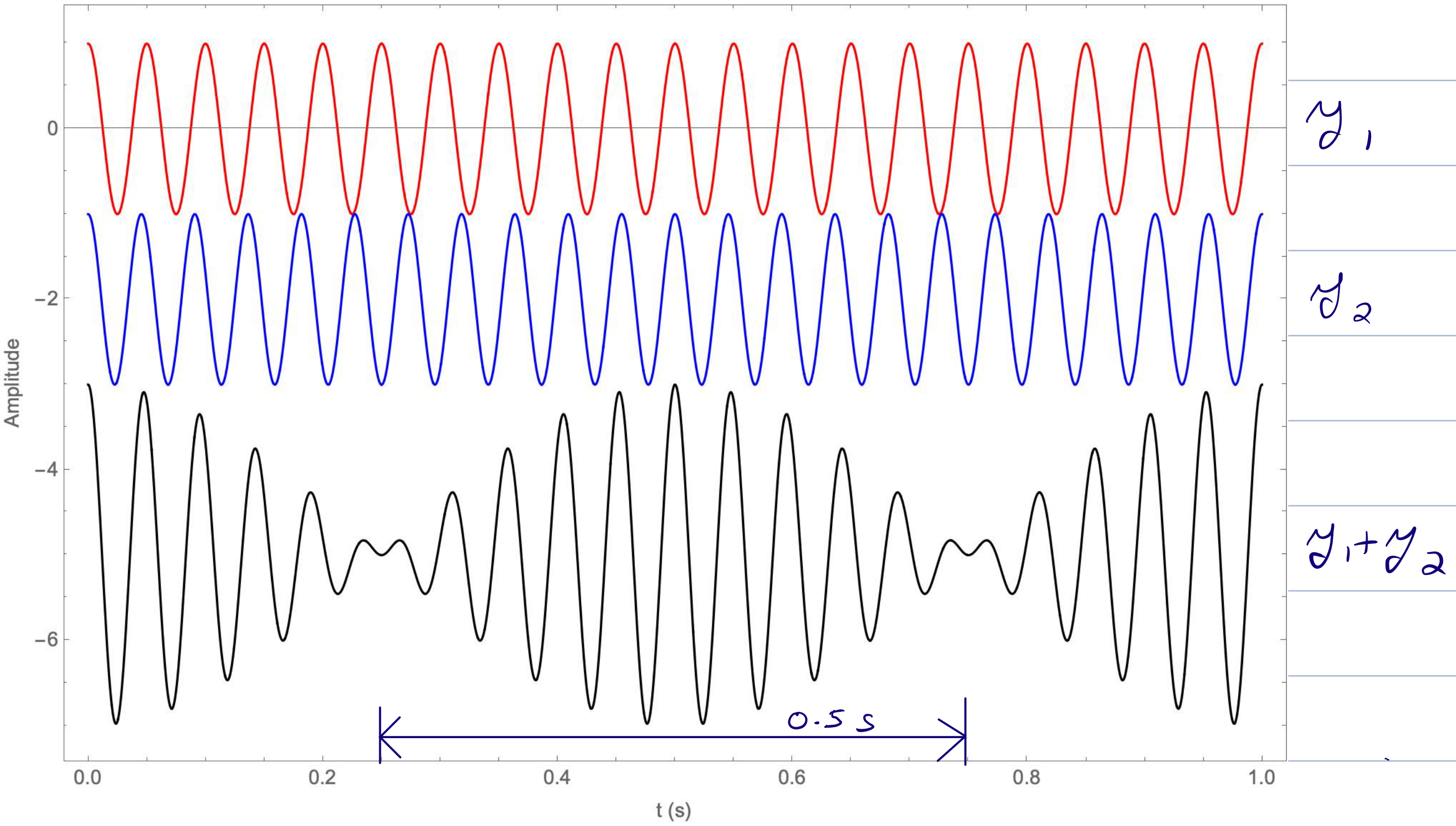
16.7 Beats

Two sources with different frequencies. (Mathematica demo)



$$f_1 = 20Hz \qquad f_2 = 22Hz \qquad \Delta f = 2Hz$$

Net result: Amplitude oscillates with a "Beat" frequency $f_{beat} = |f_2 - f_1|$.

This is useful for detecting small differences.

Example: tuning: A = 440 Hz

suppose string is tuned to 339 Hz.

You will hear "beats" at 1 Hz telling you it's slightly out of tune.

Example: Ch16-beats-2.pdf.

Mathematical interlude: (not on test)

$$y_1 = A\sin(k_1 x - \omega_1 t)$$

$$y_2 = A\sin(k_2 x - \omega_2 t)$$

$$y_{total} = y_1 + y_2.$$

Simplify: Let $x = 0$.

$$y_{total} = -A\left[\sin(\omega_1 t) + \sin(\omega_2 t)\right]$$

Trig trick: note

$$\sin\theta_1 + \sin\theta_2 = 2\cos\left(\frac{\theta_1 - \theta_2}{2}\right)\sin\left(\frac{\theta_1 + \theta_2}{2}\right)$$

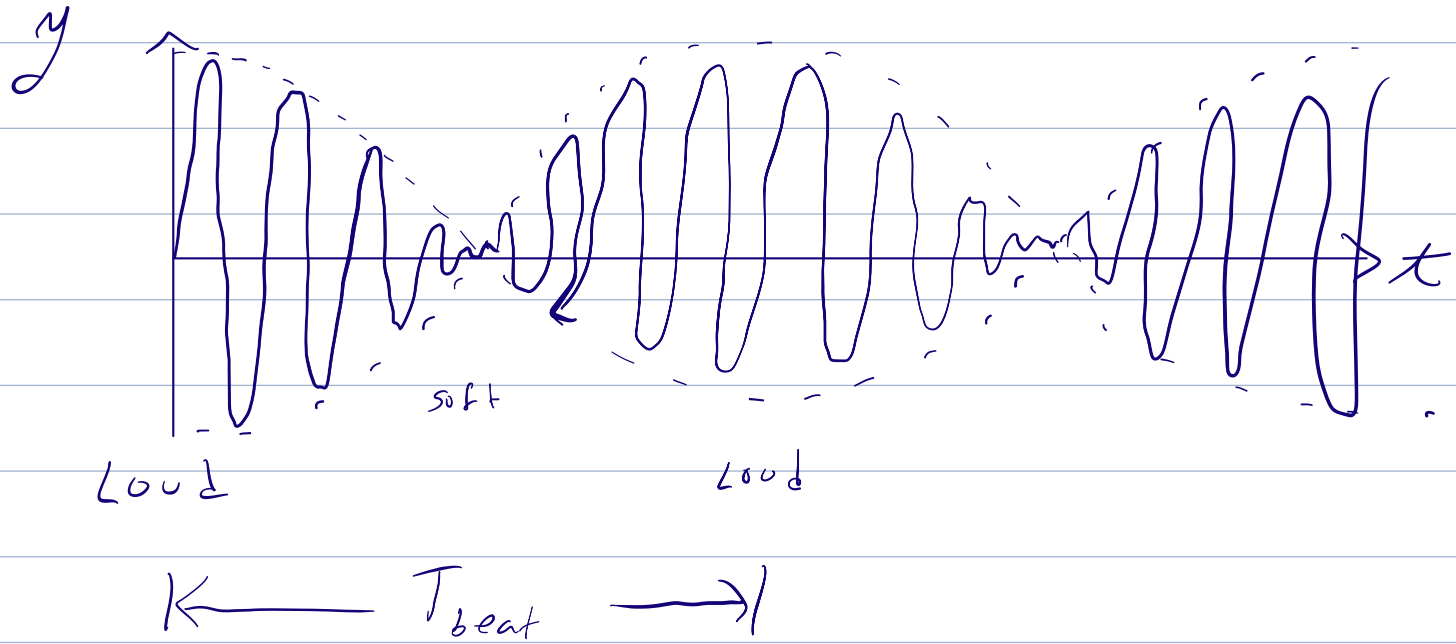
Proof: Expand RHS

$$\sin(a+b) = \sin a\cos b + \cos a\sin b$$

$$\cos(a-b) = \cos a\cos b + \sin a\sin b.$$

write it all out, get

$$y_{total} = \underbrace{-2A\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)}_{\text{slowly varying amplitude}}\ \underbrace{\sin\left(\frac{\omega_1 + \omega_2}{2}t\right)}_{\text{fast oscillation}}$$



$$f_1 = \omega_1 / 2\pi \qquad f_2 = \omega_2 / 2\pi$$

$$f_{beat} = |f_2 - f_1|$$

$$T_{beat} = \frac{1}{f_{beat}}.$$

How? Want $\cos\left(\frac{\omega_1 - \omega_2}{2} T_{beat}\right) = -1$

$$\frac{\omega_1 - \omega_2}{2} T_{beat} = \pm\pi$$

$$\frac{2\pi}{2}(f_1 - f_2) T_{beat} = \pm\pi$$

$$T_{beat} = \frac{1}{|f_1 - f_2|}$$