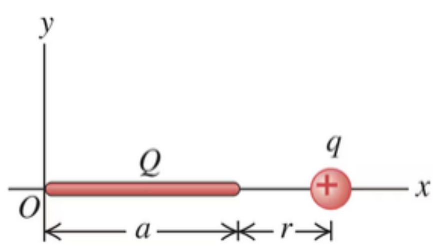
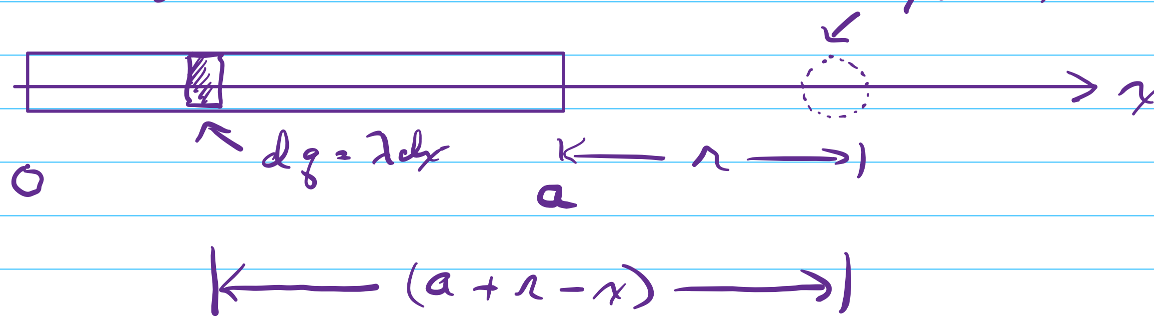
21.79 •• CALC Positive charge Q is distributed uniformly along the x-axis from $x = 0$ to $x = a$. A positive point charge q is located on the positive x-axis at $x = a + r$, a distance r to the right of the end of Q (Fig. P21.79). (a) Calculate the x- and y-components of the electric field produced by the charge distribution Q at points on the positive x-axis where $x > a$. (b) Calculate the force (magnitude and direction) that the charge distribution Q exerts on q. (c) Show that if $r \gg a$, the magnitude of the force in part (b) is approximately $Qq/4\pi\epsilon_0 r^2$. Explain why this result is obtained.

Figure P21.79



Problem 21.79



we will later put charge q at point P.

Rod: charge Q, density $\lambda = \dfrac{Q}{a}$

(a) dE = electric field due to dq, all in $+x$ direction.

$$dE = \frac{k\,dq}{(a+r-x)^2} = \frac{k\lambda\,dx}{(a+r-x)^2}$$

$$E = \int_{x=0}^{a} \frac{k\lambda\,dx}{(a+r-x)^2}$$

Let $u = a+r-x$

$du = -dx$

Limits: $x=0 \Rightarrow u = a+r$

$x = a \Rightarrow u = r$

$$E = \int_{a+r}^{r} \frac{k\lambda(-du)}{u^2} = \frac{k\lambda}{u}\Big|_{a+r}^{r}$$

$$E = \frac{k\lambda}{r} - \frac{k\lambda}{a+r}$$

simplify? Put over common denominator

$$E = \frac{k\lambda(a+r) - k\lambda r}{r(a+r)} = \frac{k\lambda a}{r(a+r)} = \boxed{\frac{kQ}{r(a+r)}}$$

$$\boxed{\vec{E} = \frac{kQ\,\hat{\imath}}{r(a+r)}}$$

(b) Force $\vec{F} = q\vec{E} = \dfrac{kQq\,\hat{\imath}}{r(a+r)}$

(c) If $r \gg a$, then $a+r \approx r$, so

$\vec{F} = \dfrac{kQq}{r^2}\hat{\imath}$, i.e. the rod looks like a point charge from far away.