Ch 16 – Interference

Mathematical Interlude – not on test or final

Given

$$y_1 = A \sin(kx_1 - \omega t)$$
$$y_2 = A \sin(kx_2 - \omega t)$$

} Two waves travelling different distances

$$y_t = y_1 + y_2$$

Trig Trick:

$$\sin\theta_1 + \sin\theta_2 = 2\sin\left(\frac{\theta_1+\theta_2}{2}\right)\cos\left(\frac{\theta_2-\theta_1}{2}\right)$$

$\therefore$

$$y_t(x,t) = 2A\sin\left(k\left(\frac{x_1+x_2}{2}\right) - \omega t\right)\cos\left(k\left(\frac{x_2-x_1}{2}\right)\right)$$

use $k = \frac{2\pi}{\lambda}$ and $x_2 - x_1 = \Delta x$, $x_{avg} = \frac{x_1+x_2}{2}$

$$y_t(x,t) = \underbrace{2A\sin(kx_{avg} - \omega t)}_{\text{regular travelling wave}}\ \underbrace{\cos\left(\frac{2\pi}{\lambda}\frac{\Delta x}{2}\right)}_{\text{constant, depends on } \frac{\Delta x}{\lambda}}$$

Constructive Interference:

When is amplitude of $y_t(x,t)$ maximum?

when $\cos\left(\pi\frac{\Delta x}{\lambda}\right) = \pm 1$, or $\boxed{\Delta x = m\lambda}$

or ... phase difference $= \Delta\phi = k\Delta x = \frac{2\pi}{\lambda}\Delta x = 2\pi m$.

Destructive Interference:

When is amplitude of $y_t(x,t)$ minimum?

when $\cos\left(\pi\frac{\Delta x}{\lambda}\right) = 0$, or $\boxed{\Delta x = (m+\frac{1}{2})\lambda}$

phase difference $= \Delta\phi = \frac{2\pi}{\lambda}\Delta x = (2m+1)\pi = (\text{odd})\pi$.