Ch24-cylindrical-capacitor

Calculate the capacitance of a cylindrical capacitor. The capacitor consists of an inner cylindrical conductor of radius Ri, and an outer cylindrical conductor of radius Ro. The cylinders are both of length L. (L is considered "large" so we don't have to worry about end effects.) A charge +Q is placed on the inner cylinder, and a charge -Q is placed on the outer cylinder. What is the capacitance of this arrangement? (This is based on the Geiger Tube example in Ohanian, Ch. 25, problem #29.)

Plan:

1. Draw a diagram, including the electric field direction.
2. Calculate the electric field between the cylinders using Gauss's Law.
3. Calculate the voltage difference by integrating E.
4. Use Q = C V to calculate the capacitance.

Data:

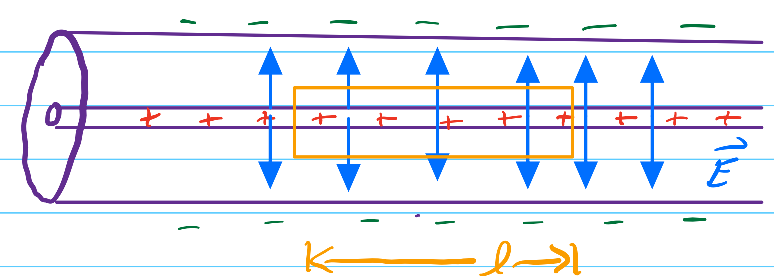
$R_i = 2.5 \times 10^{-5}$ m

$R_o = 2.5 \times 10^{-2}$ m

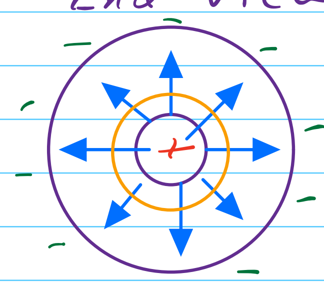
$L = 0.1$ m

$Q = 8 \times 10^{-10}$ C

1. Side View



End View



2. Use Gauss's Law on a cylindrical surface of radius r and length l (shown in orange.)

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \underbrace{\int_{\text{end caps}} \vec{E}\cdot d\vec{A}}_{0} + \int_{\text{curved part}} \vec{E}\cdot d\vec{A}$$

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$$\Phi = E \int_{\text{curved part}} dA = E(2\pi r l)$$

$Q_{inside} = \lambda l$, where $\lambda = \frac{Q}{L}$

$$\therefore \quad \Phi = Q_{inside} / \epsilon_0$$

$$E(2\pi r l) = \frac{Q l}{L \epsilon_0}$$

$$\boxed{E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}}$$

3 $\Delta V = -\int_{R_i}^{R_o} \vec{E} \cdot d\vec{l}$, let $d\vec{l}$ be radial, parallel to $\vec{E}$.

$$\Delta V = -\int_{R_i}^{R_o} \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r} dr = \frac{-\lambda}{2\pi\epsilon_0} \ln\left(\frac{R_o}{R_i}\right)$$

Note the $-$ sign: $\vec{E}$ points from high potential (the inner wire) to low potential (the outer wire).

$$\Delta V = -\frac{Q}{L} \cdot \frac{1}{2\pi\epsilon_0} \ln\left(\frac{R_o}{R_i}\right)$$

4. Use $Q = C|\Delta V|$ to find C.

$$\boxed{C = \frac{Q}{|\Delta V|} = \frac{2\pi\epsilon_0 L}{\ln(R_o/R_i)}}$$

In this example,

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$$C = \frac{2\pi\,(8.85\times10^{-12}\,F/m)\,(0.1\,m)}{\ln\left(\frac{2.5\times10^{-2}\,m}{2.5\times10^{-3}\,m}\right)} = \boxed{8.05\times10^{-13}\,F}$$

In the text, see examples 22-6, 23-10, and 24.4, which work out the same problem.