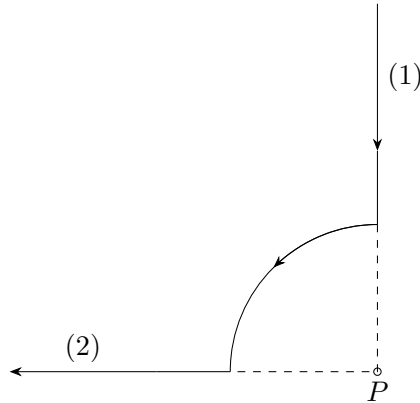
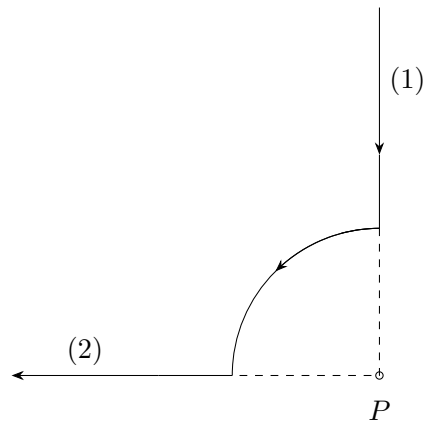


Problem 3: (30 pts.) A rigid segment of wire is formed into the shape shown in the following figure, and carries a current $I = 3.00\text{ A}$ in the direction shown. The arc shown is one quarter of a circle, and has a radius of $R = 0.400\text{ m}$. The straight segments each have length $L = 1.200\text{ m}$. The ultimate goal is to calculate the magnetic field at point P , located at the center of the arc.

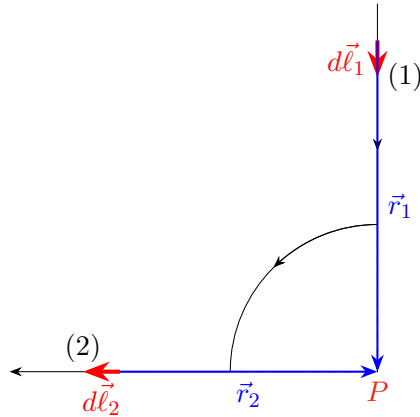


- a. (5 pts.) Consider a small segment $d\vec{\ell}$ of wire (1) a distance y above point P . What is the magnetic field $d\vec{B}$ at point P due to that small segment? Show your work clearly.
- b. (5 pts.) What is the magnetic field at point P due to the entire wire segment (1)?
- c. (5 pts.) What is the magnetic field at point P due to the entire wire segment (2)? (You do not need to re-derive anything – you may re-use results from (a) and (b) as appropriate.)



- d. (15 pts.) Use the **Biot-Savart law** to calculate the magnetic field **vector** at point P due to the circular arc. Remember that a small segment of circular arc of length $d\ell$ can be written as $Rd\theta$. You *must* use the Biot-Savart law to get any credit for this part of the problem. (As in the previous part, the current is 3.00 A and the radius of the arc is 0.400 m.) Show your work clearly.

Problem 3: (30 pts.) A rigid segment of wire is formed into the shape shown in the following figure, and carries a current $I = 3.00\text{ A}$ in the direction shown. The arc shown is one quarter of a circle, and has a radius of $R = 0.400\text{ m}$. The straight segments each have length $L = 1.200\text{ m}$. The ultimate goal is to calculate the magnetic field at point P , located at the center of the arc.



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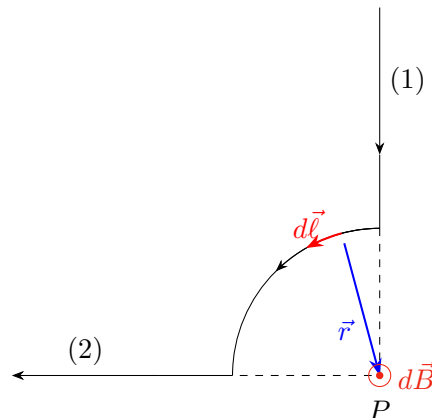
$$d\vec{B}_1 = \frac{\mu_0}{4\pi} \frac{Id\vec{\ell}_1 \times \hat{r}_1}{r_1^2} = \boxed{0}, \text{ since the angle between } d\vec{\ell}_1 \text{ and } \hat{r}_1 \text{ is } 0.$$

- b. (5 pts.) What is the magnetic field at point P due to the entire wire segment (1)?

$$\vec{B}_1 = \int_{(1)} d\vec{B}_1 = \boxed{0}. \text{ The cross product is 0 for all parts of wire segment (1), so the total field due to segment (1) is 0.}$$

- c. (5 pts.) What is the magnetic field at point P due to the entire wire segment (2)? (You do not need to re-derive anything – you may re-use results from (a) and (b) as appropriate.)

$$\boxed{0}. \text{ The angle between } d\vec{\ell}_2 \text{ and } \hat{r}_2 \text{ is } 180^\circ, \text{ so the cross product } d\vec{\ell}_2 \times \hat{r}_2 \text{ is 0. Thus the integral over segment (2) gives 0.}$$



- d. (15 pts.) **Use the Biot-Savart law** to calculate the magnetic field **vector** at point P due to the circular arc. Remember that a small segment of circular arc of length $d\ell$ can be written as $Rd\theta$. You *must* use the Biot-Savart law to get any credit for this part of the problem. (As in the previous part, the current is 3.00 A and the radius of the arc is 0.400 m.) Show your work clearly.

By the right hand rule, the cross product $d\vec{\ell} \times \hat{r}$ points out of the page, so $d\vec{\ell} \times \hat{r} = d\ell \hat{k}$. All points on the arc are the same distance $r = R = 0.400$ m away from point “P”.

$$\begin{aligned}
 d\vec{B} &= \frac{\mu_0}{4\pi} \frac{Id\vec{\ell} \times \hat{r}}{r^2} \\
 d\vec{B} &= \frac{\mu_0}{4\pi} \frac{Id\ell}{R^2} \hat{k} \\
 \vec{B} &= \int_{arc} dB \hat{k} = \int_{\theta=\pi/2}^{\pi} \hat{k} \frac{\mu_0}{4\pi} \frac{I(Rd\theta)}{R^2} = \hat{k} \frac{\mu_0 I}{4\pi R} \int_{\pi/2}^{\pi} d\theta \\
 \vec{B} &= \hat{k} \frac{\mu_0 I}{4\pi R} \cdot \frac{\pi}{2} = \frac{\mu_0 I}{8R} \cdot \hat{k} \\
 \vec{B} &= \frac{(4\pi \times 10^{-7} \text{ Tm/A}) \times (3.00 \text{ A})}{8 \times 0.400 \text{ m}} \hat{k} = \boxed{1.18 \times 10^{-6} \text{ T } \hat{k}}
 \end{aligned}$$