

Magnetic Field due to Helmholtz Coils

In[17]:= **Clear["Global`*"]**

$$B[x_] := \frac{\mu_0 i R^2}{2 (R^2 + x^2)^{3/2}}$$

(* Field due to a single coil of radius R a distance 'x' away *)

In[19]:= **Bleft[x_, s_] := B[x + s / 2]**

Bright[x_, s_] := B[x - s / 2]

Btot[x_, s_] := Bleft[x, s] + Bright[x, s]

Evaluate Derivatives at the center

In[22]:= **Simplify[D[Btot[x, R], x]]**

Out[22]=

$$48 i R^2 \left(\frac{R - 2 x}{2 (5 R^2 - 4 R x + 4 x^2)^{5/2}} - \frac{\frac{R}{2} + x}{(5 R^2 + 4 R x + 4 x^2)^{5/2}} \right) \mu_0$$

In[23]:= **% /. x -> 0**

Out[23]=

0

In[24]:= **Simplify[D[Btot[x, R], {x, 2}]]**

Out[24]=

$$768 i R^2 x \left(\frac{-R + x}{(5 R^2 - 4 R x + 4 x^2)^{7/2}} + \frac{R + x}{(5 R^2 + 4 R x + 4 x^2)^{7/2}} \right) \mu_0$$

In[25]:= **% /. x -> 0**

Out[25]=

0

In[26]:= **Simplify[D[Btot[x, R], {x, 3}]]**

Out[26]=

$$3840 i R^2 \left(\frac{R^3 - 6 R x^2 - 4 x^3}{(5 R^2 + 4 R x + 4 x^2)^{9/2}} + \frac{-R^3 + 6 R x^2 - 4 x^3}{(5 R^2 - 4 R x + 4 x^2)^{9/2}} \right) \mu_0$$

In[27]:= **% /. x -> 0**

Out[27]=

0

In[28]:= **Simplify[D[Btot[x, R], {x, 4}]]**

Out[28]=

$$23040 i R^2 \left(\frac{-3 R^4 + 16 R^3 x - 32 R x^3 + 16 x^4}{(5 R^2 - 4 R x + 4 x^2)^{11/2}} + \frac{-3 R^4 - 16 R^3 x + 32 R x^3 + 16 x^4}{(5 R^2 + 4 R x + 4 x^2)^{11/2}} \right) \mu_0$$

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In[29]:= % /. x -> 0 n(* The 4th derivative is not zero at the center. *)
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Out[29]=
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$$-\frac{27\,648\,i\,R^6\,\mu_0}{625\,\sqrt{5}\,(R^2)^{11/2}}$$

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In[36]:= B[0]
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Out[36]=
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0.000020944
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Plot the Field

Pick some plausible numbers.

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In[30]:= R = 0.15; μ0 = 4 π 10-7; i = 5.0;
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In[31]:= Btot[x, s]
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Out[31]=
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$$\frac{7.06858 \times 10^{-8}}{\left(0.0225 + \left(-\frac{s}{2} + x\right)^2\right)^{3/2}} + \frac{7.06858 \times 10^{-8}}{\left(0.0225 + \left(\frac{s}{2} + x\right)^2\right)^{3/2}}$$

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In[32]:= BHC = Btot[0, R] (* Max value at center of Helmholtz Coil configuration *)
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Out[32]=
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0.0000299725
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In[33]:= Bmax = 2 * B[0] (* Max value for 2 overlapping coils *)
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Out[33]=
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0.0000418879
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In[42]:= Coil[x_, s_] :=
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Piecewise[{{Bmax / BHC, Abs[Abs[x] - s / 2] < 0.01}}, 0] (* Draw coils *)
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In[41]:= Manipulate[Plot[{Coil[x*R, s*R], Bleft[x*R, s*R] / BHC,
  Bright[x*R, s*R] / BHC, Btot[x*R, s*R] / BHC}, {x, -1.5, 1.5},
  PlotRange -> {0, Bmax / BHC}, PlotPoints -> 600, ImageSize -> Scaled[0.9],
  LabelStyle -> Larger,
  AxesLabel -> {"x/R", "B / BHC"},
  PlotLegends -> {"Coils", "Left", "Right", "Total"},
  PlotStyle -> {Thick, Automatic, Automatic, Automatic}, Exclusions -> None],
  {{s, 2.0}, 0, 2.0, Appearance -> "Labeled"}]

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Out[41]=

