## Experiment 2

## Kinematics of a Freely Falling Body

### 2.1 INTRODUCTION

In this experiment, you will record and analyze the parabolic trajectory of a ball undergoing projectile motion.

The purpose of the experiment is to analyze the horizontal and vertical positions of the ball as a function of time, and to compare your results to the theoretical predictions for an object undergoing a constant linear acceleration. You will make and analyze graphs of position vs. time and velocity vs. time to determine the acceleration of the ball. Along the way, you will learn how to plot and analyze data in LoggerPro, and how to compare your experimental results (including uncertainties) to theoretical predictions.

### 2.2 DATA ACQUISITION

For this experiment, you need a record a video of an object thrown in a parabolic arc.
To capture a video, slide your computer along the table so it is aligned with the meter stick mounted at the other end of the workbench. If the camera is not visibly protruding from the top of the computer, press downwards on the recessed area along the top edge and it will spring upwards.

Open LoggerPro and select Insert $\rightarrow$ VideoCapture. After you have done so, a window will open that looks like Figure 2.1.

Open the Set Up Camera menu, and select a resolution of $1280 \times 720$ pixels. In the Options


Figure 2.1: The video capture window.
menu, select Video Capture Only and change the time to 5 seconds. Additionally, check the box labelled Pictures With Standard Analysis.

If adjustments need to be made to improve your video quality, these can be done via the Camera Settings $\rightarrow$ Advanced Settings menu.

One member of your group should stand near the end of the table, and practice throwing the ball in a parabolic arc, ensuring that the ball does not leave the top of the frame of the video.

LoggerPro is unable to interpret distances within the video you will record, so it is important that there be an item of known length situated in the plane of the ball's arc. A meter stick has been mounted at the end of the bench to serve this purpose. The partner responsible for throwing the ball needs to ensure that the path of the ball and the meter stick should remain equidistant from the camera during the entirety of the flight.

Hit the Start Capture button to begin recording. After a few seconds, your video will be
displayed. Inspect the first frame of the video to ensure that you can clearly identify the two ends of the meter stick. If you are unable to do so, you must record another video.

### 2.3 DATA ANALYSIS

To begin your analysis, click on the button in the lower right corner of your video, which will open a menu on the right-hand side. Next, set the scale as follows: Click on the set scale button $\omega$, then click on one end of the mounted meter stick and drag the cursor to its opposite end. Enter the size of the object in meters, and hit OK. It is important to do this step carefully. If you are unable to see the ends of your scale object clearly, you may need to shoot a fresh video.

Use the slider at the bottom of your video to locate the first frame after which the ball has left your hand. Use the left and right arrow buttons to fine tune your frame selection.
Finally, click the ${ }^{-\quad-}$ button and then move the cursor to the center of the ball. Notice that the coordinates of the ball ( $\mathrm{x}, \mathrm{y}$ ) appear on the data table in the LoggerPro window, and also on a graph of position versus time. Click the mouse button, and the video will automatically advance one frame. Continue this process until the ball disappears from the bottom of the frame, or is just about to hit the edge of the table. You have now successfully tracked the position of the ball as a function of time.

If you have any errant points, you will be able to strike them out. Click on the data table, then highlight the relevant rows in the data table and select Edit $\rightarrow$ Strike Through Data Cells. Alternatively, you can select the top arrow icon on the right hand side of the video window, click on a bad data point, and hit the Delete key.

NOTE: If the ball leaves the top of the frame at any time during the experiment, you must record another video.

### 2.3.1 Using Logger Pro to perform Graphical Data Analysis

1. The default graph that LoggerPro has produced includes plots of both the horizontal and vertical positions of the ball as a function of time. For now, we will analyze the vertical component of its motion. The x-position plot can be removed by clicking midway on the vertical axis of the graph. A box will pop up within LoggerPro containing a selection of variables that you might wish to plot. Select the appropriate variable to plot the y-position versus time.
2. Note that the chart is the current "Active Element" on the worksheet. That is, there are little squares around the graph. You can use them to move or resize the graph.
3. Nearly every aspect of a graph can be changed. To edit a graph, right click anywhere
inside the graph. Begin by setting the scale: Select Autoscale $\rightarrow$ Autoscale from 0 . (You can also double click directly on the X or Y axis to change the scales.) Experiment with various options until you are comfortable using LoggerPro.

Fortunately, the default settings are usually sensible and you usually don't have to worry about the formatting details.
4. If you want to add more graphs, use Insert $\rightarrow$ Graph. To change which columns are plotted, right-click on the graph, pick Graphing Options, click on the Axis Options tab, and choose the source of the $x$ - and $y$-axis data.

### 2.3.2 Curve-Fitting

The objective of curve-fitting is to determine the function that best fits your data and to relate the constants (also called coefficients) of the function to the physical parameters of the experiment.

A curve-fitting program achieves this by finding the coefficients that minimize the spread between individual experimental data points and the smooth curve of the function. The program reports the best-fitting coefficients and the uncertainties in those coefficients. An excellent fit would have uncertainties that are very small compared to the absolute value of the coefficients; a bad fit would have large uncertainties. The details of how these coefficients and uncertainties are calculated is not important here.
In this experiment, the primary quantity we are interested in determining is the gravitational acceleration, $g$. Since our ball is in free fall, we expect that its height, $y$, as a function of time, should be given by the free fall equation:

$$
\begin{equation*}
y=y_{0}+v_{0} t+\frac{1}{2} g t^{2}, \tag{2.1}
\end{equation*}
$$

where $y_{0}$ is the initial position, and $v_{0}$ is the initial velocity of the ball. Your goal is to determine whether Equation 2.1 adequately describes your experimental data, and, if so, the best estimates for the values $y_{0}, v_{0}$, and $g$. Finally, you will compare your experimental results to the expected values.

## Vertical Position Versus Time: Quadratic Fit

Now, you will fit your data with a polynomial of degree 2, i.e., of the form

$$
\begin{equation*}
y=A t^{2}+B t+C \tag{2.2}
\end{equation*}
$$

where $A B$, and $C$ are constant coefficients.
Select Analyze $\rightarrow$ Curve Fit and select the Quadratic equation. In the upper right corner of the "Curve Fit" box, you can either select a "Manual" or "Automatic" fit. You should usually select "Automatic," and then click Try Fit.

While still in the "Curve Fit" dialog box, check the "Time offset" box. In the small graph box, click and highlight the region of interest, starting with your first data point. This accounts for the time lag between the beginning of the video and the moment that the ball leaves your hand, yielding the equation:

$$
\begin{equation*}
y=A\left(t-t_{0}\right)^{2}+B\left(t-t_{0}\right)+C \tag{2.3}
\end{equation*}
$$

This is equivalent to setting $t=0$ to the first measurement of the position of the ball Click Try Fit. If the results look reasonable, select OK. Discuss briefly whether this is a good qualitative fit. A box will pop up that summarizes the fit parameters, including uncertainties. If the uncertainties are not visible, double-click on the box and select "Show Standard Error".

By comparing the coefficients in the theoretical free fall equation (Equation 2.1) with those in the fitted equation (Equation 2.3), find the values (and uncertainties) of the initial position $y_{0}$, the initial velocity $v_{0}$, and the acceleration due to gravity $g$. Record your results, including uncertainties, being careful to use the correct number of significant figures. Does your result for the acceleration due to gravity agree with the expected value? Include uncertainty analysis. Do your values for $v_{0}$ and $y_{0}$ make physical sense?

Record your value for the acceleration due to gravity on the blackboard. Ensure to include the uncertainty (rounded to the appropriate number of significant figures) and the units.

## Vertical Velocity Versus Time

In lecture, you will learn that, if $y$ is the position along a line and $t$ is the time, then the velocity at time $t$ is given by

$$
\begin{equation*}
v=\frac{d y}{d t} . \tag{2.4}
\end{equation*}
$$

In this experiment, the velocity $v_{i}$ in frame $i$ is well-approximated by

$$
\begin{equation*}
v_{i}=\frac{y_{i+1}-y_{i-1}}{t_{i+1}-t_{i-1}} . \tag{2.5}
\end{equation*}
$$

LoggerPro has this velocity calculation programmed in, but the approximation needs to be modified to reflect the relatively low number of data points you have collected. To do this, use menu item File $\rightarrow$ Settings for (filename), and set the Number of Points for Derivative Calculation to 3 .

Change the $y$-axis of your graph so that you are now plotting y-velocity versus time. Given your knowledge of linear motion (Equations. 2.1 and 2.4, write the expected theoretical
relationship between velocity, initial velocity, and acceleration, and include it in your lab writeup.

Examine this equation and determine which type of curve fit would be useful to extract the relevant physical quantities. Note that the first and last velocity points are likely to be erroneous due to the way that LoggerPro selects points. If this is the case with your data, these points can be excluded from your analysis by clicking and dragging only the points you wish to include.

Perform this fit, and use it to determine the acceleration due to gravity, $g$. Does this value agree with the expected value? Include uncertainty analysis.

## Analysis of Horizontal Motion

Since gravitational acceleration is in the vertical direction, we expect the horizontal acceleration of the ball to be zero. Repeat the above analysis using the horizontal positions and velocities of the ball to check whether the horizontal acceleration is consistent with the expected value of zero.

## Analysis of Class Data

Once each group has recorded their value for the acceleration due to gravity on the blackboard, record these results into a spreadsheet. Analyze this data set using the multiple trials method that you learned in the previous experiment. That is, determine the mean, standard deviation, and uncertainty of the mean of the class results. For this analysis, you can ignore the uncertainties in the individual values for the acceleration due to gravity. Do the aggregate results of the class agree with the expected value for $g$ ?

Are the three values for $g$ that you determined in today's experiment consistent with one another? Is your horizontal acceleration consistent with zero? How do you decide whether the different values agree? If there is a meaningful difference between your values, do you have any ideas what might have gone wrong?

Finally, discuss any ways in which the experiment or write-up could be improved.

