Experiment 4

Energy Conservation and the Spring

4.1 INTRODUCTION

In this experiment, you will study how Newton's laws and energy conservation apply to a stretched spring. You will gain experience using model-fitting and uncertainty analysis to compare experimental results with a theoretical model.

4.2 SPRING FORCE

If a spring is stretched or compressed by an amount x, it imposes a force

$$F_{\mathrm{e}} = -kx.$$

Here k is the *spring constant*, a fixed value for any given spring; the minus sign reminds us that the force is in the direction opposite that which the spring has been stretched or compressed; and the subscript e is used because another name for spring force is *elastic* force.

In this experiment, you will be hanging masses off the end of a vertically mounted spring. The masses consists of a metal hanger and additional metal pieces which can rest on the hanger. Rather than directly measuring the stretch of the spring, you should measure the vertical position of the bottom of the hanger, y. The force is then

$$F_{\mathrm{e}} = -k(y - y_0),$$

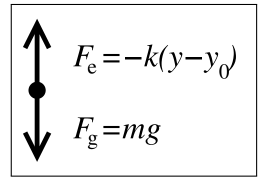

Figure 4.1: Free body diagram for the mass hanging from the end of the spring. Notice that if the spring is stretched downward, $y < y_0$, so F_e as drawn here is a positive number.

where y_0 is the position where the hanger would be if there were no mass.[1]

A free body diagram for the mass is in figure 4.1. If the mass is at rest, $a = 0$, then

$$\begin{aligned} \sum F &= ma \\ F_e - F_g &= 0 \\ -k(y - y_0) - mg &= 0 \\ y &= \left(-\frac{g}{k}\right) m + y_0 \qquad (4.1) \end{aligned}$$

The negative sign in Equation 4.1 arises because increasing the mass causes the hanger to be closer to the table top, which is a lower value of y.

4.2.1 Measurements

Your work bench will be supplied with a spring hanging from a support above the table top. *Caution:* to avoid damaging the spring, do not stretch it beyond 65 cm.

Hang a mass on the spring and note the mass m. Don't forget to include the mass of the hanger. Plug the motion detector into the DIG 1 port in the `LabQuest Mini` interface and open `LoggerPro`. Lay the motion detector on the table, ensuring that the circular metallic speaker is centered under the mass. Wait for the spring to become stationary and hit the collect button to produce five seconds of data. Since the spring isn't moving, the plot of position versus time in `LoggerPro` should be approximately horizontal. Choose Analyze: Statistics to determine the mean position of the mass and record this value. This is the position y of the bottom of the mass hanger relative to the position of the motion detector. The smallest mass you should use should be enough to visibly stretch the coils. Typically, this is at least 15 g. The largest mass should stretch the spring until it is no closer than 20 cm from the motion detector. This detector is unable to reliably echo-locate objects closer than this. Measure y for about six to eight masses more-or-less evenly spread between these limits.

[1] Since the hanger itself has mass, and the equilibrium position is defined when the system has no mass, and you use the bottom of the hanger to measure positions, you can't directly measure the equilibrium position, y_0. You need to find it by analyzing measurements as described in §4.2.2.

4.2.2 Analysis

Enter your masses, m, and positions, y, into `Logger Pro`. Give your data columns meaningful names and units. You can do that with the `Data`→`Manual Column Options` menu item, or by directly clicking on the `x` and `y` labels at the top of the data table.

Plot y *vs.* m; that is, make a plot with m on the x-axis and y on the y-axis.

Use `Analyze`→`Curve Fit` and select a a straight line fit to your data. Make sure the font is large enough in the fit box: Right click within the fit box and select `Linear Fit Options` and then `Appearance` and select a larger point size, such as 14. Make sure the graph is zoomed appropriately to show all the features of your graph. Usually the autoscale function (either the large `A` on the toolbar or the `Analyze`→`Autoscale`→`Autoscale` menu option works well.

From the slope and y-intercept of the fit, find k and y_0 using Equation 4.1. Don't forget to include uncertainties in these calculations.[2] Since the slope of the line is proportional to k^{-1},

$$\frac{\sigma_k}{k} = \frac{\sigma_{\text{slope}}}{\text{slope}} \qquad \Rightarrow \qquad \sigma_k = \frac{\sigma_{\text{slope}}}{\text{slope}}\, k.$$

Be sure to also comment on whether the straight line is a good *qualitative* fit to your data. Does the straight line capture the general trend of your data, or is is missing something important?

4.3 SPRING ENERGY

There are three types of energy in the mass-spring system:

$$\text{Kinetic Energy:} \qquad K = \tfrac{1}{2}mv^2 \tag{4.2}$$

$$\text{Elastic Potential Energy:} \qquad U_{\text{e}} = \tfrac{1}{2}k(y - y_0)^2 \tag{4.3}$$

$$\text{Gravitational Potential Energy:} \qquad U_{\text{g}} = mgy \tag{4.4}$$

The total mechanical energy at any point is the sum of these,

$$E = K + U_{\text{e}} + U_{\text{g}}.$$

In this part of the experiment, continue to use the same mass-and-spring system as before. You will release the mass from some height, measure positions and velocities using `LoggerPro`, and calculate and analyze its energy at three different positions—the bottom, the middle, and top.

[2] If the box on the `LoggerPro` graph which shows the slope and intercept of the line does not give uncertainties on these quantities, right-click on the box, pick `Linear fit options`, select `Show uncertainties`, and click `OK`.

4.3.1 Data Acquisition

1. Attach a mass which is about 2/3 of the maximum used for the first part of the experiment. Measure the equilibrium point, y_{eq}, where it sits at rest. Pull the mass down from equilibrium until its bottom is about 20 cm above the motion detector. Let go of the mass and press the Collect button on `LoggerPro` immediately afterwards.

   Do not alter the position of the horizontal support bar during this process.

   Your goal is to get a nice smooth oscillating position graph with at least one full cycle with no significant glitches. If your graph has any problems, try again or ask your instructor for help.

2. Click on the position graph and find a point near the top of the motion. Record the position y_{top}. Look in the data table and record the velocity v_{top} *at the same time.*

3. Similarly, record the position y_{mid} and velocity v_{mid} at a point near the equilibrium position.

4. Lastly, record the position y_{bottom} and velocity v_{bottom} at a point near the bottom of the motion.

   As always, leave your set-up intact. Don't take it apart until you have successfully completed the analysis.

4.3.2 Analysis

Contemplate the kinetic, spring potential, and gravitational potential energies at the top, middle, and bottom of the oscillation. Briefly describe in words how you expect each term to vary as the mass progresses from the top of its motion to the bottom. Record your expectations in your report so that you can compare them to the results of your analysis below.

Use Equations 4.2 through 4.4 to calculate the kinetic energy, elastic potential energy, and gravitational potential energies at top, middle, and bottom of the oscillation. Add them up to find the total energy at each of these three points. Present your results in a neat table listing the values of K, U_{e}, U_{g}, and total energy E at each of the three points. (You may find it handy to do these calculations in Excel.)

How do the energies at each position compare with your qualitative expectations recorded above?

If energy is conserved in your system, the total energy should be the same at each point. It is difficult to estimate the uncertainties in the positions and velocities you have determined, so it is also difficult to estimate the uncertainty in the energies. However, given your results (and ignoring uncertainties) does it seem that energy is conserved? What factors do you think contributed most to the uncertainty in the energy? Explain your reasoning, including any specific observations you may have made today that support your conclusion.

As always, discuss sources of uncertainty and ways in which the experiment or write-up could be improved.