Experiment 6

Rotational Energy

6.1 INTRODUCTION

In this experiment, you will study energy conservation in a system that has both rotational and translational motion. The experiment itself is straightforward: the gravitational potential energy of a falling mass is transformed into *translational* kinetic energy of the falling mass and *rotational* kinetic energy of a rotating platter. The challenge in the experiment lays in connecting the quantities of interest for energy calculations, such as the speed of the falling mass and the angular speed of the disk, to the quantities actually measured in the experiment.

The apparatus is shown in Figure 6.1. A large platter rotates on a low-friction bearing. Actually, what we refer to as "the platter" is made of two disks, held together by friction. A small metal hub is attached to the platter. A string is wound around the hub. The string is attached, via a pulley, to a hanging mass. As the mass falls, the platter rotates. There is a small flag attached to the rotating platter; it passes through a photogate once per revolution of the platter. By measuring the amount of time it takes the flag to pass through the photogate, the speeds of the platter and the hanging mass can be inferred. By counting the number of times the flag has passed through the photogate, the number of revolutions can be inferred, so the length of the string unwound from the hub, and hence the height of the hanging mass, can be calculated.

6.2 THEORY

The rotational kinetic energy of the rotating platter is

$$K_{rot} = \frac{1}{2}I\omega^2,$$

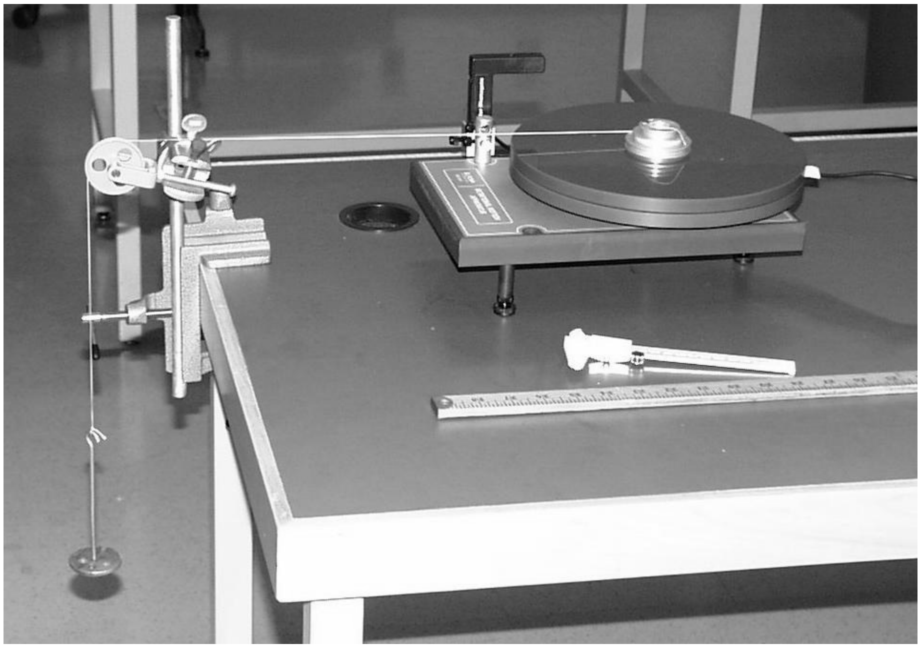

Figure 6.1: Apparatus for the rotational energy experiment.

where I is the moment of inertia of the spinning platter and ω is its angular velocity. The kinetic energy of translation of the falling mass is

$$K_{trans} = \frac{1}{2}mv^2,$$

where m is its mass and v is its velocity. The gravitational potential energy of the falling mass is

$$U_{\mathrm{g}} = mgy.$$

Since there is no non-gravitational work in this system, energy is conserved,

$$K_{rot_{\mathrm{i}}} + K_{trans_{\mathrm{i}}} + U_{\mathrm{g_i}} = K_{rot_{\mathrm{f}}} + K_{trans_{\mathrm{f}}} + U_{\mathrm{g_f}}$$

or, more generally,

$$K_{rot} + K_{trans} + U_{\mathrm{g}} = E_{\mathrm{total}} = \mathrm{constant}.$$

Your goal is to calculate each of these terms, K_{rot}, K_{trans}, and U_{g}, as the mass falls, and to check whether E_{total} truly remains constant.

6.3 MEASUREMENTS

6.3.1 Moment of inertia

The moment of inertia of a solid disk of mass m and radius r is

$$I = \frac{1}{2}mr^2.$$

The spinning platter actually consists of two gray disks along with a the hub around which the string is wound. You can pull the disks off the spinning apparatus. Measure the mass of each disk on a scale and add them together to find the total mass m_{disk}. Find its radius, r_{disk}, by measuring its diameter and dividing by 2. It will be helpful to work in standard units (kilograms, meters, seconds); convert all measurements to these units as you work through this experiment. Calculate the moment of inertia of the disk using $I = \frac{1}{2}m_{\text{disk}}r_{\text{disk}}^2$. The hub, because it is light and close to the rotation axis, has a very small moment of inertia, around $0.000009\,\text{kg}\,\text{m}^2$, and can be ignored in the calculation of I.

6.3.2 Angular size of the flag

See Figure 6.2 for an overhead view of the apparatus. Find the angular size of the flag, θ_{flag}. This is the ratio of the width of the flag to the radius of a circle measured at the position of the photogate sensor. This radius is used because this is where the measurement of the flag will actually be made. Measure w_{flag} and $r_{\text{photogate}}$ and then calculate

$$\theta_{\text{flag}} = \frac{w_{\text{flag}}}{r_{\text{photogate}}}.$$

Angular size, θ_{flag} will have units of radians as long as w_{flag} and $r_{\text{photogate}}$ were measured using consistent units.

For your uncertainty analysis below, you will need an estimate of the uncertainty in the size of the flag, $\sigma_{w_{\text{flag}}}$. In principle, you could very precisely measure the flag width with a caliper. However, this is difficult since it isn't sturdy enough to tightly clamp the caliper down. In this case, a ruler is the best option. Look carefully at the ruler and its smallest subdivisions and use this information to estimate your uncertainty.

6.3.3 Radius of the hub

In your first experiment, described below, use the smallest part of the hub. Measure its radius, r_{hub}; you will need this later. The easiest way to do this is to measure its diameter with a caliper and then divide by 2. Measure this where the string is would around the hub (not at its rim).

Again, you will need an estimate of the uncertainty of this measurement. What is actually needed is the distance at which the string comes off of the hub, the uncertainty of which

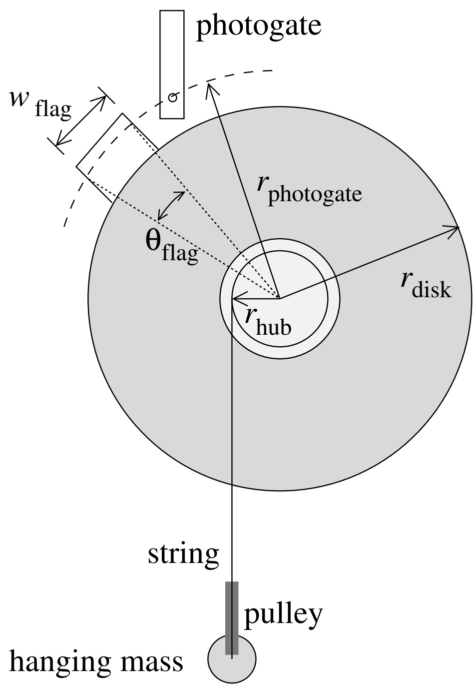

Figure 6.2: Overhead view of the apparatus.

is dominated by the width of the string and its precise position as it is wrapped around the hub. Since the caliper is more precise than the ruler, this uncertainty is likely to be smaller. Use the readout on the front of the caliper to estimate this uncertainty. How many digits does it provide?

6.3.4 Hanging mass

Measure the mass of the hanging mass. Usually these are 50 g, but it would be good to check. Don't forget to convert to kilograms.

6.3.5 Measurements as the mass falls

Line up everything carefully. The string from the platter to the pulley should be horizontal, and it should be aligned with the pulley (not coming in at an angle).

Launch `Logger Pro` then select `File`→`Open`→`Probes & Sensors`→`Photogates`→`One Gate Timer`. If a box appears that complains about not finding the probe, click `connect`. This usually fixes the problem.

In the `Photogate Distance` box, enter the angular size of the flag, θ_{flag} in radians. You can leave the units in the box as meters. When doing velocity measurements later, you will need to remember that the computer's display of "m/s" should really be "rad/s."

Carefully wind the string around the hub, minimizing overlaps.

Pres `Collect` to start data collection in `Logger Pro`.

Release the platter from rest while the flag is a short distance (say 1 cm) behind the photogate. (That is, it should not be blocking the photogate, and it should pass through the gate shortly after you release the platter. You can rotate the bottom platter independently to line it up properly.) Stop the apparatus by hand before the mass hits the floor, and then press `stop` in `LoggerPro`.

Each time the flag passes through the photogate, `Logger Pro` measures the time the flag enters the photogate and the time the flag exits the photogate, from which it calculates the Gate Time. It uses this time, along the value of θflag that you entered earlier, to calculate the angular velocity of the disk, ω, in rad/s. This is listed as "velocity" on the `Logger Pro` spreadsheet, probably with the incorrect units of m/s. You should get at least four such measurements, or more when the smallest hub radius is used.

You will need these values of ω, along with the number of times the disk has rotated—0 for the first measurement, 1 for the second, and so on—for your analysis below. We will refer to these measurements as ω_n and the rotation number as n.

6.4 CALCULATIONS

6.4.1 Energy calculations

Use `Excel` to analyze your data. Don't use a calculator. Do calculations *within* `Excel` by entering formulas into `Excel` cells. If you don't know how to do this, ask your instructor to show you. You may want to put constants used in your calculations—r_{disk}, m_2, etc.—into `Excel` cells so that you can refer to them in calculations.

Start by setting up a spreadsheet with one row per measurement and with the following eight columns:

1. Rotation number, n.

   Enter the numbers 0, 1, 2, . . . into successive rows.

2. Angular velocity, ω_n.

   Enter the measurements from `LoggerPro`.

3. Height of the hanging mass, y_n.

   Let's take the height of the mass to be zero at rotation number 0. Each time the platter makes a complete rotation, the mass falls by $2\pi r_{\text{hub}}$. (Convince yourself that this is true.) Since it is falling, its height is negative:

   $$y_n = -2\pi r_{\text{hub}} n.$$

   Fun fact: in `Excel`, you can write `PI()` for the value of π.

4. Velocity of the hanging mass, v_n.

   This is related to the angular velocity of the disk by (convince yourself this is true):

   $$v_n = r_{\text{hub}} \omega_n.$$

5. Potential energy of the hanging mass, U_{g_n}.

   This can be calculated from the height of the mass by:

   $$U_{\text{g}_n} = m g y_n.$$

   Since y_n is negative (except for $y_0 = 0$), U_{g_n} is also negative.

6. Translational kinetic energy of the hanging mass, K_{trans_n}.

   This can be calculated from the hanging mass velocity:

   $$K_{trans_n} = \frac{1}{2} m v_n^2$$

7. Rotational kinetic energy of the rotating platter, K_{rot_n}.

   This is related to angular velocity by:

$$K_{rot_n} = \frac{1}{2} I \omega_n^2.$$

8. Total energy, E_{n}.

$$E_n = U_{\mathrm{g}_n} + K_{trans_n} + K_{rot_n}$$

Compare the various energies you have calculated. Which are largest in magnitude? One of them is likely negligible; which is it? How are the other two energy terms related to each other? Qualitatively, does it seem like energy is conserved? If your E_{total} numbers show wide deviations from each other, try go back and to figure out what has gone wrong.

6.4.2 Uncertainty calculations

To make a more rigorous check of energy conservation, we need to consider the uncertainties in the energy measurements.

Hopefully you decided that the kinetic energy of the falling mas is relatively small, so we will ignore that in the uncertainty calculations.

What is the uncertainty in the potential energy of the falling mass?

Potential energy U_{g_n} is proportional to y_n, which in turn is proportional to r_{hub}. Guessing that this is the dominant uncertainty in the calculation of U_{g_n}, you can estimate the uncertainty in each U_{g_n} calculation by

$$\left|\frac{\sigma_{U_{\mathrm{g}n}}}{U_{\mathrm{g}n}}\right| = \left|\frac{\sigma_{r_{\mathrm{hub}}}}{r_{\mathrm{hub}}}\right| \qquad \Rightarrow \qquad \sigma_{U_{\mathrm{g}n}} = \frac{\sigma_{r_{\mathrm{hub}}}}{r_{\mathrm{hub}}} |U_{\mathrm{g}n}|$$

Add a column to your `Excel` spreadsheet with this calculation.

What is the uncertainty in the kinetic energy of the rotating platter?

Kinetic energy K_{rot_n} is proportional to ω_n^2, which is proportional to θ_{flag}^2, which in turn is proportional to w_{flag}^2. Guessing that this is the dominant uncertainty in K_{rot_n}, you can then estimate the uncertainty in each K_{rot_n} calculation by

$$\left|\frac{\sigma_{K_{rot_n}}}{K_{rot_n}}\right| = 2\left|\frac{\sigma_{w_{\mathrm{flag}}}}{w_{\mathrm{flag}}}\right| \qquad \Rightarrow \qquad \sigma_{K_{1n}} = 2\left(\frac{\sigma_{w_{\mathrm{flag}}}}{w_{\mathrm{flag}}}\right) K_{rot_n}$$

Add a column to your `Excel` spreadsheet with this calculation.

Thinking about the uncertainties

Which uncertainty is most important? How does the *variation* in E_{total} compare with the largest uncertainty calculation? Is the difference between the largest and smallest E_{total} less than 3 times the largest uncertainty? This is a good, if crude, check on energy conservation.

6.4.3 Make a plot

Have `Excel` make a single scatter plot showing U_{g_n}, K_{rot_n}, and K_{trans_n} versus n.

6.5 THAT WAS FUN, LET'S DO IT AGAIN

Now that you have your spreadsheet all set up, it is easy to repeat the experiment with other parameters. Save a copy of your spreadsheet before doing this, and use a fresh copy (with the equations in place) to analyze your next set of data.

Use a larger hub radius and/or a larger hanging mass. Repeat the measurements and calculations as described above. You should only need to change a few data numbers in your spreadsheet. Is energy conserved this time? Did this work out better or worse than your first experiment.

As always, write up a short summary of your results in your lab writeup, along with a discussion of sources of uncertainty and suggestions for improving this experiment.