Experiment 8

Periodic Motion: The Pendulum

8.1 INTRODUCTION

In this experiment, you will study the motion of a physical pendulum. You will model that motion and explore to what extent, if any, the physical pendulum exhibits simple harmonic motion. Finally, you will look at the role of damping in this system.

8.2 THEORY

For an object undergoing simple harmonic motion, the position θ as a function of time is given by

$$\theta(t) = A\cos(\omega t + \phi)\ , \tag{8.1}$$

where A is the amplitude of the motion, ω is the angular frequency, and ϕ is the phase angle. For simple harmonic motion, A and ϕ are set by the initial conditions, and ω is determined by the physical properties of the system. In particular, ω is independent of A and ϕ.

In this experiment, you will look at the motion of a physical pendulum. This pendulum consists of a long thin rod attached to a pulley. A brass mass is attached about 1/4 of the way from the end. The angular position of the pulley can be read by `LoggerPro` so you can directly measure $\theta(t)$.

For small oscillations, the angular frequency of the pendulum is given by

$$\omega = \sqrt{\frac{Mgd}{I}}\ , \tag{8.2}$$

where M is the mass of the pendulum, I is the moment of inertia of the pendulum about the axis of rotation, and d is the distance from the axis of rotation to the center of mass.

In this experiment, you don't have an easy way to measure I, M, or d (since it's not easy to measure the properties of the pulley), so we will focus on more general issues of periodic motion instead.

8.3 EXPERIMENT

8.3.1 Modeling Periodic Motion

In this portion of the experiment, you will measure $\theta(t)$ and then try to fit Eq. 8.1 to the data in order to determine A and ω.

Getting Started

1. Start `LoggerPro`. Open up the experiment file `Probes and Sensors -> Rotary Motion Sensor -> Rotary Motion angular.cmbl`.
2. Next, you need to adjust the sensor so that it doesn't re-zero itself at the beginning of each run. Select `Experiment-> Set Up Sensors -> LabQuest Mini:1`, and then click on `Dig/Sonic1`. Make sure there is no check mark next to `Reset on Collect`. If there is, click on it. Make sure the units are set to `rad`. Close the LabQuest Mini window.
3. Unfortunately, the rotary sensor frequently loses track of where the appropriate location for 0 radians should be. Fortunately, it's easy to manually re-zero before each trial. From the main menu, select `File -> Settings` and make sure the box for `Show Zero Button on toolbar` is checked. Let the pendulum hang straight up and down at rest, and then press the `Zero` button. The probe should now read 0 radians. Note that the sensor will still need frequent re-zeroing throughout the experiment.
4. Use the `Experiment ->Data Collection` menu to set the experiment Duration to 10 seconds and the Sampling Rate to 50 points per second.
5. Gently swing the pendulum back and forth and press `Collect`. When it is done collecting data, you should have a roughly sinusoidal curve. It will probably be convenient to re-adjust the scaling. One way to do that is to use the `Autoscale` icon on the tool bar (located just to the left of the zoom icons).

   Note that the curve will be vertically shifted away from 0 due to the zeroing problem of the sensor. That's ok. You will adjust for this in your curve-fits below.
6. Save your work so far.

Experiments

Now that everything is working, it's time to take some data and analyze it.

1. Rezero the rotary sensor. Rotate the pendulum until the initial angle is approximately 1.0 radians. Release the pendulum from rest and press the `Collect` button at the same time. Note that there is a considerable time delay between when you press `Collect` and when the computer actually starts collecting data.

2. Try to fit your data to a function like Eq. 8.1. Click on the "Angle" graph so it is the active graph. Select `Analyze ->Curve Fit`, and then select the `Cosine` function:

   `A*cos(Bt+C)+D`

   Comparing this to Eq. 8.1, `A` is the amplitude, `B` is the angular frequency ω, `C` is the phase angle ϕ, and `D` is the offset needed because the sensor tends to lose track of exactly where zero should be. Press `Try Fit`.

   If necessary, manually adjust the various coefficients to improve the fit. For example, you can add or subtract multiples of 2π to reduce ϕ to a value in the range $[0 : 2\pi]$. (You can use the tiny arrow next to the ϕ box to set the increment to 6.2832 so that you can add or subtract multiples of 2π with a single click.)

   You should also make sure that `A` and `B` (*i.e.* A and ω) are positive. (You can do this all manually, or you can manually adjust the coefficients, and then select `Automatic` again and have `LoggerPro` try again.) If `LoggerPro` has trouble fitting your data, you may just do a manual fit. When you are satisfied, press `Ok`, save your work, and print out your graph.

3. Use the calculated angular frequency to determine the period of the pendulum. Does the value seem reasonable? What is the uncertainty in that period? Also, does the value for the amplitude seem reasonable? If you started with about 1.0 radians, then the amplitude should be in that range as well. Don't worry about the phase just yet. You should also have observed that the amplitude of each swing slowly decreases with time. This is due to damping; you'll look at that more carefully below.

4. In order to make sense of the phase, we have to deal with the time delay between when you press the `Collect` button and when `LoggerPro` actually starts to record data. Select `Experiment -> Data Collection` and select the tab for `Triggering`. Check the box for `On Keyboard`. Now, when you press the `Collect` button, `LoggerPro` will not start recording data until you press a key on the keyboard.

5. Start the pendulum swinging again. Press `Collect` and then try to press a key on the keyboard at the appropriate time in the swing such that the phase ϕ will be close to zero. (That is, the pendulum will be at its maximum.)

   If all went well, `LoggerPro` should report a phase close to zero. (Values within ± 0.5 of 0 are fine.) If it doesn't, make sure that `LoggerPro` has selected a positive value for A and ω, and that ϕ has been reduced to the range $[0 : 2\pi]$. Make any manual adjustments necessary. Don't be a perfectionist, but don't be afraid to do a few trials. After all, each one takes much less than a minute.

Print out your graph and describe in words what the pendulum was doing at the start of your graph.

6. Next, try to get a graph such that ϕ is approximately $\pi/2$. Again, print out your graph and describe in words what the pendulum was doing at the start of your graph.

Cleaning Up

From now on, we won't care about the phase, so go back and uncheck the triggering box. Also, from now on, we won't care about the sign of the amplitude or frequency.

8.3.2 Simple Harmonic Motion?

Is the pendulum a simple harmonic oscillator? One of the hallmarks of simple harmonic motion is that the period is independent of the amplitude. Is that the case for your pendulum?

Try it and see. Determine the period as a function of amplitude. Try different amplitudes covering as wide a range as is practical. In order to minimize the effects of damping, you may wish to press the `Stop` button after only a few oscillations have completed, rather than waiting for the full 10 seconds.

The format of this section is up to you. Your goal is to answer the question "Is the pendulum a simple harmonic oscillator?" using whatever data you choose to acquire. Use whatever charts, tables, or graphs you deem appropriate. Describe your findings clearly. *Take the time to do this part well.*

8.3.3 Damping

As noted above, the amplitude of each swing slowly decreases with time. For the simplest sort of damping, and for small amplitudes, a reasonable approximation of the motion is given by

$$\theta(t) = A_0 + Ae^{-t/\tau} \sin(\omega t + \phi) \ , \tag{8.3}$$

where τ is the characteristic damping time. The offset A_0 has to be included to account for the zeroing problems of the rotary sensor.

1. Try it. Change the experiment time to 30 s. Set the pendulum swinging with a moderate amplitude (about 1.0 radians) and press `Collect`.

2. Try fitting this data to the damped function in Eq. 8.3. In the `Curve Fit` menu, select the `Damped Harmonic` function. For this experiment, the signs of A and ω do not matter, and you may ignore the phase ϕ.

3. Once your are satisfied, print out your results. In a rough sense, τ is the time it takes for the oscillation to decay "significantly." For example, after $\ln(2)\,\tau$ seconds, the oscillation amplitude will be half of the original value, and after about 5τ, it will be less than 1% of the original value. Does your value for τ look reasonable?

As always, discuss sources of error and ways in which the experiment or write-up could be improved.