Experiment 9

Resonance

9.1 INTRODUCTION

If a sinusoidally varying force is applied to a harmonic oscillator with very little damping, the mass will initially respond with seemingly random movement back and forth, but will eventually settle down to sinusoidal oscillations at the same frequency as the driving force. The amplitude of the oscillations will be quite small, however, unless the frequency of the driving force is near the natural frequency of the oscillator, that is, the frequency the oscillator would have if there were no driving force. The oscillations build up dramatically as the frequency gets closer and closer to the natural frequency. This phenomenon is called resonance. The amplitude of the oscillations will reach a maximum at a frequency known as the "resonance frequency".

In this lab, you will observe the steady-state motion of a driven oscillator, and will investigate the dependence of the steady-state amplitude on the driving frequency.

9.2 DATA ACQUISITION

The oscillator consists of a glider mounted between two springs on a horizontal air track. A "flag" mounted on top of the glider will pass through a photogate that will be used to measure the period of the motion. Two magnets attached to the glider will provide some damping.

Throughout this lab, you do not have to calculate or record uncertainties.

1. Measure the mass m of the glider, including the flag and the two magnets.
2. One of the springs attached to the glider on your air track has its other end connected to the piston of a mechanical vibrator. For now, make sure that the lever on the

mechanical vibrator is in the LOCKed position so that you do not damage the vibrator while you move the glider by hand.

3. Turn the air blower on.

4. Carefully align the springs and the spring holders so that the springs are parallel to the track.

5. Gently push the glider so that it oscillates back and forth with a moderate amplitude (around 5 to 10 cm). Note how the amplitude of the oscillation decays only very slowly. Then, place a bar magnet on each side of the glider. This will help dampen the oscillations and hasten the approach to steady state. Note how the amplitude of the oscillation now decays somewhat more rapidly. Move the magnets around until it takes about $5 \sim 10$ oscillations for the amplitude to decay to one-half of its value. Try to keep the magnets symmetrically placed so that the glider stays balanced.

6. Note down a rough estimate of the half-life. For example, if you start with an initial amplitude of 10 cm, and it takes about 7 oscillations for the amplitude to decay to 5 cm, then you would record the half-life as 7 T. (The T means the period of oscillation, but you should leave your answer symbolically in terms of the letter T.)

7. Gently push the glider so that it oscillates back and forth with a moderate amplitude. Measure the period of the oscillation with `LoggerPro`. Use the file `Probes and Sensors -> Photogates -> Pendulum Timer`. If the period appears to change significantly, check to make sure that the track is level, the springs are parallel to the track, the blower is on maximum, and the glider moves freely. Once you are satisfied that all is in order, measure the period of the motion and calculate the frequency. This is the *natural frequency* f_0 of your glider.

8. Now unlock the piston by sliding the lever to the UNLOCK position. You must be very careful never to turn on the vibrator while it is still locked. Conversely, it is important to LOCK the vibrator again when you have finished using it to protect it from damage when the glider is being moved by hand.

9. Turn on the Digital Function Generator-Amplifier (DGFA). It should be set on the 0.1-10 Hz range. The amplitude knob should be turned off (fully counter-clockwise), then rotated clockwise about 3/4 turn. Adjust the frequency knob until your glider's natural frequency is displayed. Let the glider reach steady-state and make note of its amplitude.

10. Make small adjustments to the frequency and let the glider reach steady-state again. Continue fine-tuning the frequency until you obtain the largest possible amplitude at steady-state. Don't be a perfectionist, however. It suffices to get close enough to the resonance peak that you can be sure you have a sensible setting for the amplitude. If the oscillations become so large that either spring bunches up enough to scrape

against the track, turn down the amplitude control a bit. If the oscillations remain small, turn the amplitude control up a bit. Record the approximate frequency at which the amplitude resonance occurs. Compare it to the natural frequency of your glider. Again, it is not worth trying to pin this down to better than $\pm 0.02\,\text{Hz}$, so don't spend too much time on this step.

From this point on, do not change the amplitude knob.

11. Now make a series of measurements to determine how the steady-state amplitude A depends on the driving frequency f_d.

    You should take enough data points to get a smooth curve, but don't waste time taking too much redundant data. Make sure to take a few measurements at high and low frequency. (Change the frequency range, if necessary.) The extreme frequencies should be far enough from resonance that the amplitude is relatively small compared to the amplitude at resonance. *If you plot your data as you go along, it will be* much *easier to estimate when you have enough data.*

12. Make a plot of steady-state amplitude A *vs.* driving frequency f_d. Comment on your results. Do they look qualitatively like a resonance curve?

9.2.1 Curve-Fitting

The theoretical shape for the resonance curve is given by

$$A = \frac{F_0}{\sqrt{(k - m(2\pi f_d)^2)^2 + b^2(2\pi f_d)^2}} \tag{9.1}$$

where F_0 is the maximum amplitude of the forcing, k is the spring constant, m is the oscillating mass, f_d is the driving frequency, and b is the damping constant (more on this later).

Use `LoggerPro` to fit your data to the resonance curve. Use the `Define Function` button. Entering the formula can be a little tricky. One way to enter it is as follows. Put in your measured value for m. You enter π by literally typing `pi`. This formula assumes you have used the name f for the column with the frequency.

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F0/sqrt((k-m*(2*pi*f)^2)^2+b^2*(2*pi*f)^2)
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`LoggerPro` will find values for the fit parameters F_0, k, and b.

1. Is the fit reasonable? Does it capture most of the qualitative features of your data?

2. Do the values for the parameters make sense? *You are not looking for perfect correspondence, just a good sense of whether the fitted values are plausible.*

   Hints:

(a) You used a similar spring before in the spring lab, so you can check back there to help estimate whether the fitted value for k is plausible. Since there are two springs, the effective value is twice that of a single spring.

(b) You can estimate F_0 by looking at the oscillations Δx of the plunger and using your known value of the spring constant to estimate the magnitude of the resulting force $|F_0| = k\Delta x$.

(c) Finally, you can make an estimate of b by using the known mass m, natural frequency f_0, and the half-life estimate you made during the initial set-up. For example, suppose you determined that it took about 7 oscillations for the amplitude to decay to half of its initial value, and that the natural frequency of your glider is f_0. Then you can *estimate* b by

$$b = 2\ln(2)\frac{m}{t_{1/2}} \approx \frac{(1.386)\, m\, f_0}{7}$$

Again, this is just a rough *estimate* to see if the magnitude of the fitted value returned by LoggerPro is plausible. (Since b is squared in Eq. 9.1, the sign returned by LoggerPro doesn't matter.)

As always, discuss sources of error and ways in which the experiment or write-up could be improved.