Experiment 10

The Vibrating String

10.1 PURPOSE

In this experiment, you will

1. Examine the way in which the frequencies of standing waves on a stretched string depend on factors such as the mode of oscillation, the tension, the length, and the linear mass density of the string; and
2. Gain experience designing a series of experiments to test an hypothesis.

Although this experiment focuses on a particular mechanical system, many of these ideas of resonance and normal modes apply to many different oscillatory systems, including sound waves in a tube and electromagnetic waves inside a laser cavity.

10.2 INTRODUCTION

Consider a string of length L stretched between two fixed endpoints. Waves will travel down the string with a velocity $v = \sqrt{F_T/\mu}$ where F_T is the tension in the string and μ is the mass per unit length of the string.

In a sinusoidal wave, each particle of the string vibrates up and down in the transverse direction like a simple harmonic oscillator with a period T. The frequency (in cycles per second, or Hertz) is defined by $f = 1/T$, but we often use the angular frequency $\omega = 2\pi f$ in radians per second (abbreviated s^{-1}) instead. The wavelength, wave speed and frequency are related by $\lambda = v/f$. We also define the wavenumber k by $k = 2\pi/\lambda$. Then the displacement of the particle at position x along the string at time t for a wave traveling toward positive x is

$$y = y_m \sin(kx - \omega t) = y_m \sin\left(\frac{2\pi}{\lambda}x - 2\pi f t\right) \quad .$$

The constant y_m is the amplitude of the wave.

A *standing wave* can be set up on the string by two identical waves traveling in opposite directions down the string. The net displacement is

$$y(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t) \quad .$$

The trigonometric identity

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

can be used to obtain our final result, namely,

$$y(x,t) = 2y_m \sin(kx)\cos(\omega t) = 2y_m \sin\left(\frac{2\pi}{\lambda}x\right)\cos(2\pi f t) \quad .$$

In other words, a particle on the string at position x will undergo simple harmonic motion $\cos(2\pi f t)$ with an amplitude $A(x)$ that varies with position: $A(x) = 2y_m \sin(2\pi x/\lambda)$. This is illustrated in Figure 10.1. At some points, called *nodes*, the amplitude is zero; at other points, called *antinodes*, the amplitude is a maximum given by $2y_m$.

Since both fixed ends of the string must be nodes, for a given frequency only certain combinations of wave speed and string length will allow resonance to occur. When the string resonates with n antinodes showing, the wavelength of the standing wave must be $\lambda_n = 2L/n$, and the frequency, length and tension are related by

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}} \quad . \tag{10.1}$$

The mode with $n = 1$ is called the *fundamental*, and mode number n is in general called the n^{th} *harmonic.*

Equation 10.1 is the main prediction you wish to test in this experiment.

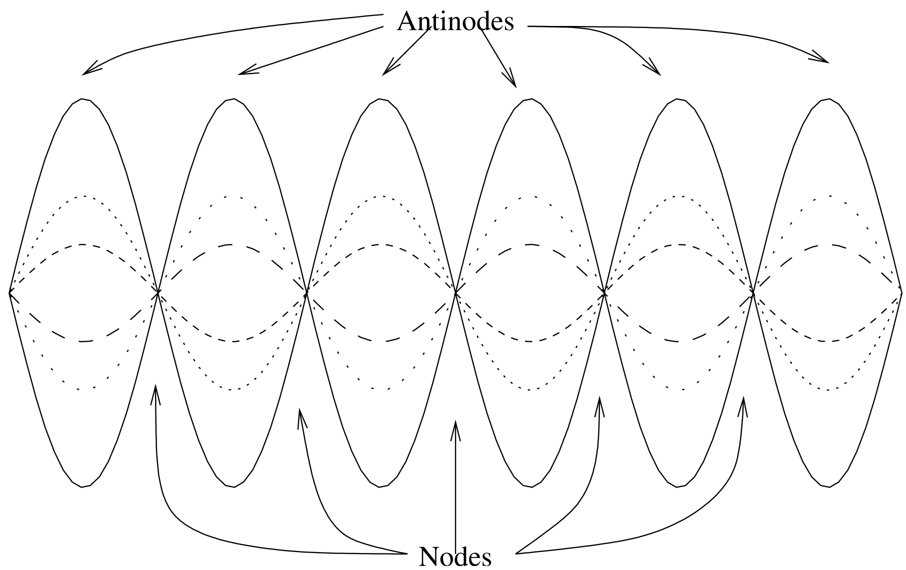

Figure 10.1: Standing waves on a string.

10.3 EXPERIMENT

10.3.1 Getting Started

The apparatus consists of a horizontal string fastened to a clamp at one end, with the other end stretched over a pulley by a mass affixed to the free end. A mechanical oscillator (a loudspeaker cone attached to a vertical plunger) applies a sinusoidal force to the string and is driven by a digital function generator. At least three different kinds of string, with different linear mass densities, will be available in the lab.

The clamped end of the string and the top edge of the pulley should be as close to the same height as possible, and that height should be adjusted so that the string sits tight in the connector on top of the mechanical oscillator without being noticeably lifted by that connector. An O-ring holds the string onto the vertical piston. On occasion, the vibrations cause this O-ring to be dislodged; if your apparatus isn't working, check to see if it has fallen off.

The oscillator can be easily damaged. **Make sure that the plunger on the oscillator is locked while you are setting up the apparatus or whenever you make adjustments to the apparatus, and unlocked before you turn on the function generator.**

The oscillator should be placed as close to the pulley as is practical, and the appropriate length to use for the "vibrating portion" of the string is the distance from the oscillator to the clamped end, *not* from the pulley to the clamp.

Connect the two input sockets on the oscillator to the GND and LOΩ terminals on the function generator. Check that the waveform switch is set to sine wave. Unlock the oscillator and make sure that the amplitude knob on the generator is at its lowest setting before you turn on the power. Increase the amplitude slightly; you can adjust it up or down as needed throughout the experiment, but you should rarely need to increase it above the 9PM setting. In general, *keep the amplitude low* to avoid non-linear effects in the string's vibrations.

As an initial trial, hang a total of 200g from the end of the string. Slowly adjust the variable frequency knob until a resonance with 2 antinodes is achieved. (Most frequencies in today's experiment will be in the 1-200 Hz range.) Fine-tune the frequency until you have made the amplitude of that resonance as large as possible. Congratulations! You have found the resonant frequency of the $n = 2$ mode.

10.3.2 Testing Equation 10.1

Your goal today is to test whether Equation 10.1 correctly predicts the oscillation frequency for a variety of conditions. Specifically, you can vary four different quantities: the mass density μ, the length L of the string that is allowed to vibrate, the mode number n, and the tension F_T. For each trial, you can also determine the resulting frequency f_n.

Design a series of experiments to investigate the dependence of the frequency in Equation 10.1 on the four quantities under your control. *Review your proposed procedure with your instructor*, and then carry out the measurements. Consider the following suggestions:

1. Perform a series of experiments in which only one of the four parameters is varied at a time. That is, keep three parameters constant and vary the remaining one. Then, perform three additional experiments in which the other quantities are varied.

2. Once you find the frequency f_n of one of the modes, Equation 10.1 should help you predict the frequencies of other modes (with different values of n). You can use this prediction as an initial estimate of where to begin; you should then fine-tune the frequency to find the largest possible amplitude, and record that fine-tuned frequency.

3. Present your results graphically. For example, Equation 10.1 predicts that f_n is proportional to n. What would you expect the shape of a graph of f_n versus n to look like? For a straight line, consider both the slope and the intercept, and include units. Do they agree with expected values, including uncertainties?

4. If you plot f_n vs. F_T, what kind of curve would you expect? Consult Equation 10.1 if you are unsure. Based on this prediction, perform an appropriate curve fit. Does this fit match your data? Compare your fit parameters to the values predicted by Equation 10.1. Don't forget to include units (these can be somewhat messy).

5. Repeat the previous step with plots of f_n vs. L and f_n vs. μ
6. It is not necessary to make dozens of measurements for each experiment. If you **plot your data as you go along**, it should be fairly clear when you have enough data to move on.
7. Be sure your data spans the full range of available conditions. Consult with your instructor if you have questions.

Additional Experimental Hints

1. Remember to *keep the amplitude low* when you are trying to fine-tune the frequency to find resonance.
2. For the experiments where you keep the tension constant, a tension of roughly 2N seems to work well.
3. For the experiments where you vary the tension, a range of 100 g to 1000 g works well. (Check the heaviest mass first to make sure nothing twists or bends under that much tension.)
4. For the experiments where you keep the number of antinodes n constant, $n = 2$ or $n = 3$ seem to work well.
5. Keep the string straight. Make sure the driver stays in line with the end posts.
6. Make sure the driver doesn't pinch the string too tightly, but also make sure the string isn't too loose. This is especially important when you are varying the length of the vibrating portion of the string. Ask your instructor if you have questions.

Since an important part of this experiment is for you to design the experimental procedure, make sure you explain clearly in your notebook what you have done.

10.4 ANALYSIS

Do your data confirm Equation 10.1? Explain your conclusions carefully.

As usual, discuss sources of error. Do not just give a long list of possible errors. Discuss what you think were the most significant ones. If you conclude that there were no significant sources of error, then write that and explain your reasoning. Some examples of questions to consider are: Do you expect some modes to give better results than others? If so, why? Do you expect some strings to give better results than others? If so, why?

Finally, discuss ways in which the experiment or write-up could be improved.