39. (II) A 75-kg trampoline artist jumps vertically upward from the top of a platform with a speed of 5.0 m/s. (*a*) How fast is he going as he lands on the trampoline, 3.0 m below (Fig. 6–37)? (*b*) If the trampoline behaves like a spring of spring constant 5.2×10^4 N/m, how far does he depress it?

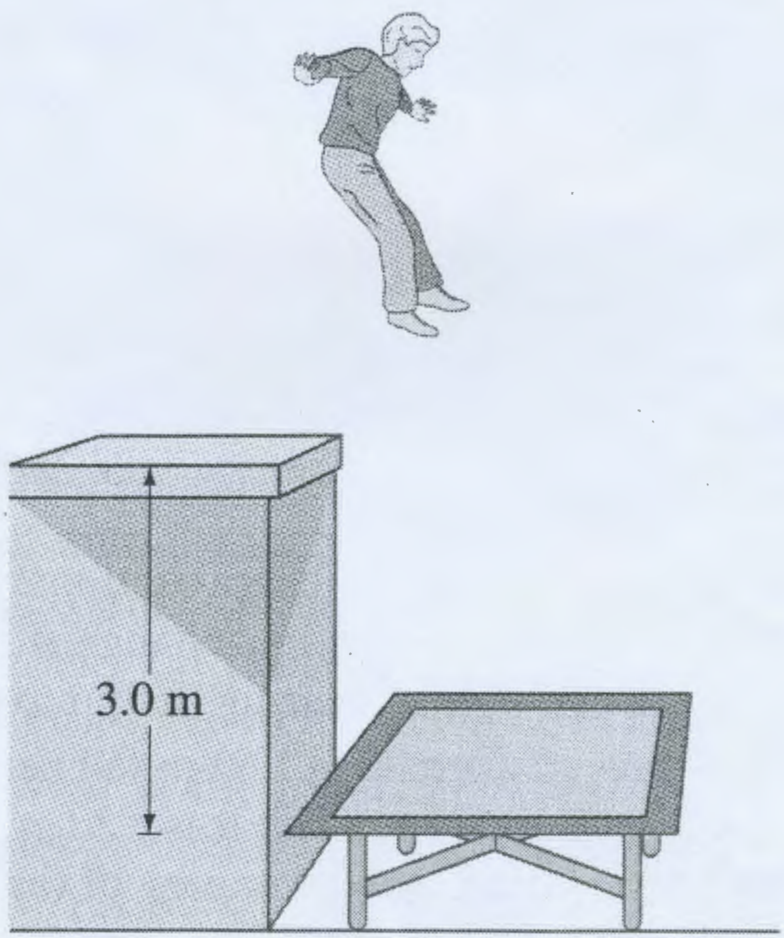
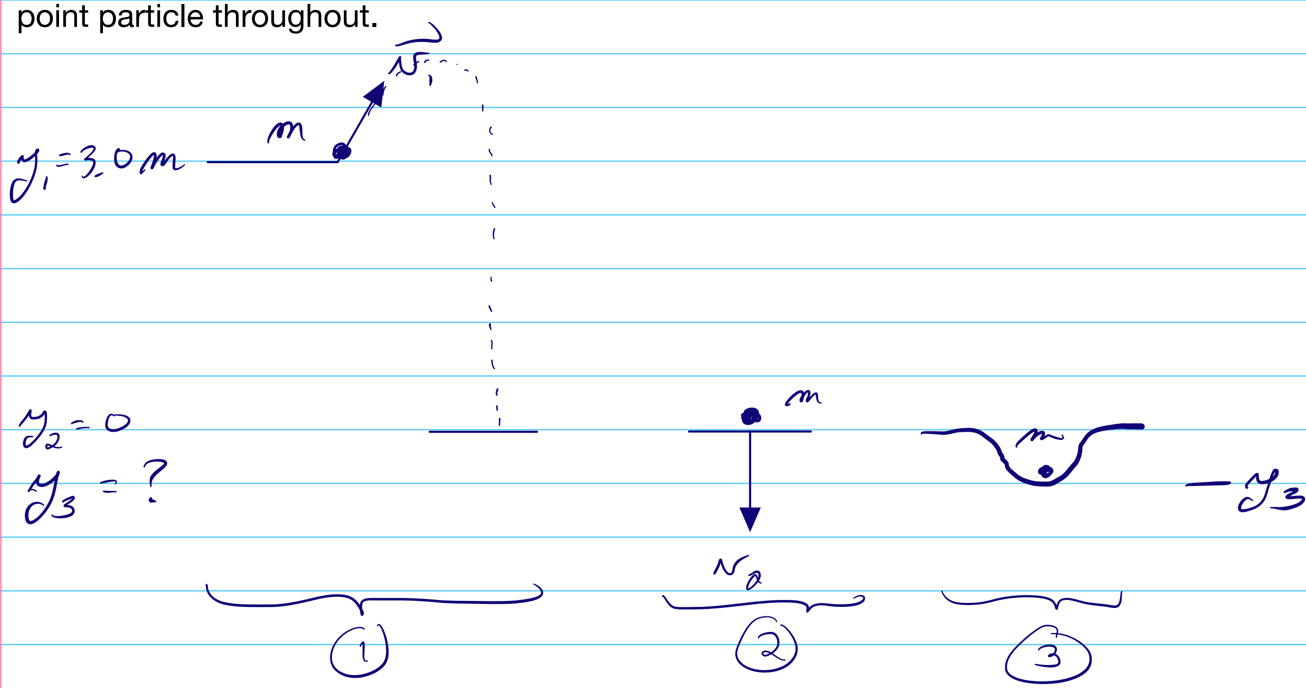


FIGURE 6–37 Problem 39.

This problem is from Giancoli's textbook, Chapter 6.

The first step is to draw a clear diagram. We can break the problem down into several distinct states: The initial jump, the time when the artist reaches the trampoline, and the final state when the trampoline is stretched the maximum amount. Treat the artist as a point particle throughout.



State 1: Artist jumps with initial speed v_1

State 2: Artists reaches trampoline with speed v_2

State 3: Artist is momentarily at rest as the trampoline is stretched the maximum amount.

Given: $y_1 = 3.0\,m$
$v_1 = 5.0\,m/s$
$m = 75\,kg$
$y_2 = 0$ = relaxed height of trampoline
$k = 5.2 \times 10^4\,N/m$

What is y_3?

Strategy: Since the spring exerts a variable force, we can't assume constant acceleration, so $\Sigma F = ma$ won't be helpful here.

Since both gravity and springs are conservative, we can use Energy conservation.

Roundabout plan: Use $E_1 = E_2$ to find speed v_2, and then use $E_2 = E_3$ to find the maximum stretch.

Direct plan: Since $E_1 = E_2 = E_3$, omit the middle step and just use

$$E_1 = E_3$$
$$K_1 + U_1 = K_3 + U_3$$

Note in picture 1, the spring is unstretched, so there is no spring potential. In picture 3, we are still on planet Earth so there is still gravitational potential energy as well as spring potential energy. Also in 3, the velocity is zero at the maximum stretch, so $K_3 = 0$.

$$K_1 + U_1 = K_3 + U_3$$
$$\frac{1}{2}mv_1^2 + mgy_1 = 0 + mgy_3 + \frac{1}{2}ky_3^2$$

Note that I took the relaxed position $y_2 = 0$.

$$0 = \frac{1}{2}ky_3^2 + mgy_3 - \left(\frac{1}{2}mv_1^2 + mgy_1\right)$$

We are now faced with a quadratic. There are no significant cancellations or simplifications, so we can plug numbers in now or later.

$$\frac{1}{2}\left(52{,}000\ \tfrac{N}{m}\right)y_3^2 + (75\ kg)\left(9.8\ \tfrac{m}{s^2}\right)y_3 - \left(\frac{1}{2}(75\ kg)(5.0\ m/s)^2 + (75\ kg)\left(9.8\ \tfrac{m}{s^2}\right)(3.0\ m)\right) = 0$$

$$\left(26{,}000\ \tfrac{N}{m}\right)y_3^2 + (735\ N)\,y_3 - 3142.5\ J = 0$$

$$y_3 = \frac{-735 \pm \sqrt{(735)^2 - 4(26000)(-3142.5)}}{2(26000)}$$

$$y_3 = \frac{-735 \pm 18093}{52000}\ m = \begin{cases} +0.334\ m \\ \text{or} \\ -0.362\ m \end{cases}$$

Since the spring is depressed and $y_3 < 0$, pick the negative root

$$\boxed{y_3 = -0.362\ m}$$