3. A piston is oscillating vertically in simple harmonic motion with an amplitude A and an angular frequency of $\omega = 2\,\mathrm{rad/s}$. A small mass $m = 0.0100\,\mathrm{kg}$ is placed on top of the piston. What is the maximum amplitude of oscillation of the system such that the mass m remains in contact with the piston throughout the motion?

3. A piston is oscillating vertically in simple harmonic motion with an amplitude A and an angular frequency of $\omega = 2\pi f = 2\,\text{rad/s}$. A small mass $m = 0.0100\,\text{kg}$ is placed on top of the piston. What is the maximum amplitude of oscillation of the system such that the mass m remains in contact with the piston throughout the motion?

The constraint that it just barely remains in contact suggests thinking about contact forces, specifically the normal force. Apply Newton's second law to the mass sitting on top of the piston:

$$\sum F = ma$$
$$F_N - mg = ma$$

If the mass just barely remains in contact, then then normal force goes to zero.

$$0 - mg = ma$$
$$-g = a$$

The object will tend to first come off the piston at the top of the oscillation. (Try it yourself and see!) The acceleration at the top is $-a_{max}$, where $a_{max} = (2\pi f)^2 A$ is the magnitude of the maximum acceleration, and A is the amplitude of the oscillation.

$$-g = -a_{max}$$
$$g = a_{max} = (2\pi f)^2 A$$
$$A = \frac{g}{(2\pi f)^2}$$
$$A = \frac{9.8\,\text{m/s}^2}{(2\,\text{rad/s})^2}$$
$$A = 2.45\,\text{m}$$