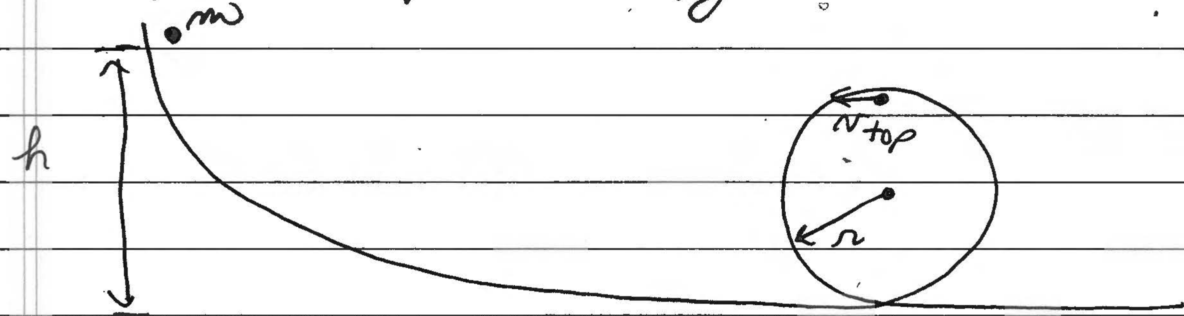
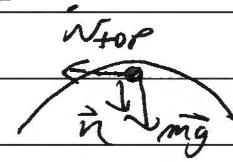
Loop-the-Loop – Energy Conservation



Release ball from rest. What is the minimum initial height h such that the ball just barely stays on the track at the top?

1st: what is v_{top}? Use $\sum F = ma$



$$-\vec{n} - m\vec{g} = -m v^2/r\ \hat{j}$$

Let $n \to 0$. $v_{top} = \sqrt{gr}$.

2nd Conserve energy

$$E_i = E_f$$

$$U_i + K_i = U_f + K_f$$

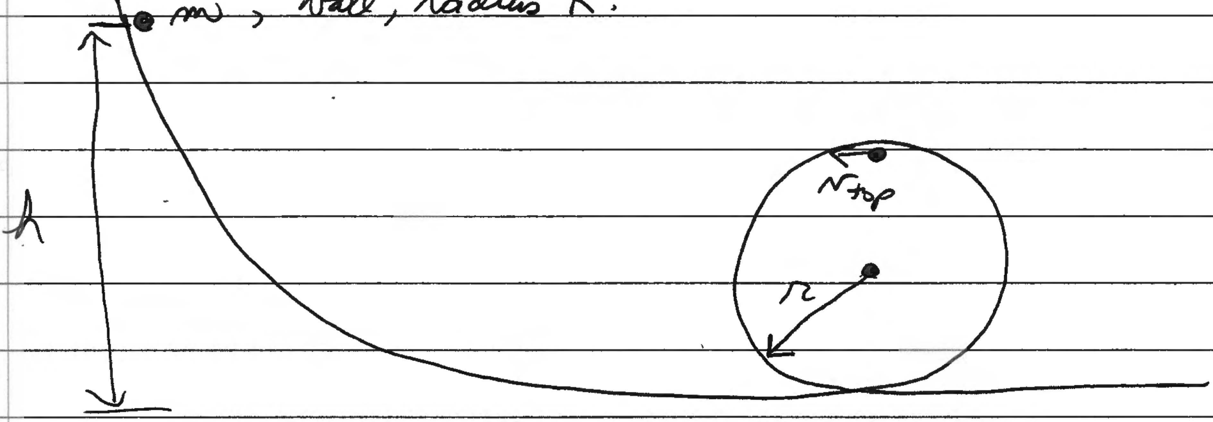
$$mgh + 0 = mg(2r) + \tfrac{1}{2} m v_{top}^2$$

$$gh = g(2r) + \tfrac{1}{2}(gr) = \tfrac{5}{2} gr$$

$$\boxed{h = \tfrac{5}{2} r}$$

$r = 0.215\,m \Rightarrow h \approx 0.538\,m.$

Loop-the-Loop: Energy Conservation / Rolling



Release ball from rest. What is the initial minimum height h such that the rolling ball just barely stays on track at the top?

1^{st}: Newton's 2^{nd} Law yields $v_{top} = \sqrt{gr}$

2^{nd} Apply energy conservation for a rolling ball

$$E_i = E_f$$

$$U_i + K_i = U_f + K_f$$

$$mgh + 0 = mg(2r) + \frac{1}{2}mv_{top}^2 + \frac{1}{2}I\omega^2$$

rolling $\Rightarrow$ $\omega = v_{top}/R$

ball $\Rightarrow$ $I = \frac{2}{5}mR^2$

just stays on $\Rightarrow$ $v_{top} = \sqrt{gr}$

$$\therefore\ mgh + 0 = mg(2r) + \frac{1}{2}m(gr) + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{gr}{R^2}\right)$$

$$mgh = mg(2r) + \frac{1}{2}m(gr) + \frac{1}{5}m(gr)$$

$$h = \left(2 + \frac{1}{2} + \frac{1}{5}\right)r = \boxed{2.7r}$$

If $r = 0.215m$, $\boxed{h \approx 0.581m}$