4. (30 pts.) On a horizontal air track, an air car of mass 0.360 kg is attached to a horizontal spring with a spring constant of 9.0 N/m. The car is oscillating back and forth in simple harmonic motion. Assume there is no damping. At time $t = 0\,\text{s}$, the car is moving through the origin with a positive velocity of $1.2\,\text{m/s}$.

a. (10 pts.) What is the magnitude of the maximum acceleration of the car?

b. (10 pts.) What is the earliest time at which that acceleration occurs?

c. (10 pts.) Now suppose that you want to add damping such that after 10 complete oscillations, the amplitude will have decreased to $1/e$ of its original value. (That is, at $t = 10T$, you want $A = A_0 e^{-1}$.) What value for the damping coefficient b will be required?

4. (30 pts.) On a horizontal air track, an air car of mass 0.360 kg is attached to a horizontal spring with a spring constant of 9.0 N/m. The car is oscillating back and forth in simple harmonic motion. Assume there is no damping. At time $t = 0$ s, the car is moving through the origin with a positive velocity of 1.2 m/s.

a. (10 pts.) What is the magnitude of the maximum acceleration of the car?

Way #1: SHM

$x = A \sin \omega t$

$v = \omega A \cos \omega t$

$a = -\omega^2 A \sin \omega t$.

Max accel $= \omega^2 A$

Max velocity $= \omega A$

$\omega = \sqrt{k/m} = 5$ rad/s

$(1.2\,m/s) = (5\,rad/s) A$

$A = 0.24\,m$

$a_{max} = \omega^2 A = 6\,m/s^2$

Way #2 Energy and $F = ma$

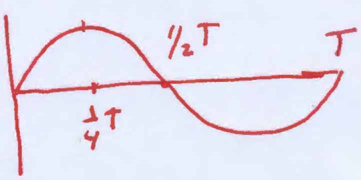
$E_i = E_f$

$\frac{1}{2} m v_i^2 = \frac{1}{2} k A^2$

$A = \sqrt{k/m}\; v_i = 0.24\,m$

$a = \frac{F}{m} = -\frac{kA}{m} = \boxed{6\,m/s^2}$

b. (10 pts.) What is the earliest time at which that acceleration occurs?



occurs when $\sin(\omega t) = 1$, or

$\omega t = \pi/2$

$\frac{2\pi}{T} t = \pi/2 \Rightarrow t = \frac{1}{4} T$

$t = \frac{1}{4} \cdot 2\pi \sqrt{\frac{m}{k}} = \boxed{0.314\,s}$

c. (10 pts.) Now suppose that you want to add damping such that after 10 complete oscillations, the amplitude will have decreased to $1/e$ of its original value. (That is, at $t = 10T$, you want $A = A_0 e^{-1}$.) What value for the damping coefficient b will be required?

$A = A_0 e^{-bt/2m} = A_0 e^{-1}$

$\therefore \frac{bt}{2m} = 1, \quad b = \frac{2m}{t} = \frac{2m}{10T} = \frac{2(0.36\,kg)}{10(1.257\,s)} = \boxed{0.0573\,\frac{kg}{s}}$