Physics 122-01—Thermodynamics and Waves
Quiz #2
Monday, March 7, 2005

Problem 1: (40 pts.) A 5.00 kg partridge is suspended from a pear tree by an ideal spring of negligible mass. When the partridge is pulled down 0.100 m below its equilibrium position and released, it vibrates with a period of 1.20 s.

Big hint: For parts (a), (b), and (c) below, you may simply treat the motion as simple harmonic motion about the equilibrium position. The fact that it is vertical rather than horizontal shifts where the equilibrium position is (and that is the focus of part (d)), but does not change the answers to parts (a), (b), or (c).

a. (10 pts.) What is its speed as it passes through the equilibrium position?

b. (10 pts.) What is its acceleration when it is 0.050 m above the equilibrium position?

c. (10 pts.) When it is moving upward, how much time is required for it to move from a point 0.0500 m below its equilibrium position to a point 0.0500 m above it?

d. (10 pts.) The motion of the partridge is stopped, and then it is removed from the spring. How much does the spring shorten?

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a. (10 pts.) What is its speed as it passes through the equilibrium position?

You could simply use $v_{\text{max}} = \omega A$, but here is a longer approach using energy conservation. The final position is at the equilibrium position, y_{eq}:

$$E_i = E_f$$
$$K_i + U_i = K_f + U_f$$
$$\frac{1}{2}m0^2 + mgy_i + \frac{1}{2}k\left(y_i - y_R\right)^2 = \frac{1}{2}mv_{\text{eq}}^2 + mgy_{\text{eq}} + \frac{1}{2}k\left(y_{\text{eq}} - y_R\right)^2$$

The equilibrium position can be obtained by setting $\sum F = 0$ at the equilibrium position.

$$F_s - mg = 0$$
$$-k\left(y_{\text{eq}} - y_R\right) - mg = 0$$
$$y_{\text{eq}} = y_R - \frac{mg}{k}$$

Or, equivalently,

$$y_R = y_{\text{eq}} + \frac{mg}{k}$$

You can plug this in to the energy expression and expand everything out, but the algebra is much simpler if you set the origin at $y_{\text{eq}} = 0$:

$$y_R = 0 + \frac{mg}{k}$$
$$mgy_i + \frac{1}{2}k\left(y_i - y_R\right)^2 = \frac{1}{2}mv_{\text{eq}}^2 + \frac{1}{2}k\left(0 - y_R\right)^2$$
$$mgy_i + \frac{1}{2}k\left(y_i^2 + y_R^2 - 2y_iy_R\right) = \frac{1}{2}mv_{\text{eq}}^2 + \frac{1}{2}ky_R^2$$
$$mgy_i + \frac{1}{2}ky_i^2 - ky_iy_R = \frac{1}{2}mv_{\text{eq}}^2$$
$$mgy_i + \frac{1}{2}ky_i^2 - ky_i\left(\frac{mg}{k}\right) = \frac{1}{2}mv_{\text{eq}}^2$$
$$\frac{1}{2}ky_i^2 = \frac{1}{2}mv_{\text{eq}}^2$$

Note that since the amplitude of the motion is $A = y_{eq} - y_i = -y_i$, this is simply the familiar result for motion of a simple harmonic oscillator. You could also simply start this problem here:

$$\begin{aligned}\frac{1}{2}kA^2 &= \frac{1}{2}mv_{eq}^2 \\ v_{eq}^2 &= \frac{k}{m}A^2 = \omega^2 A^2 \\ v_{eq} &= \omega A\end{aligned}$$

We can find k and ω from the period of oscillation T and the mass m:

$$\begin{aligned}\omega &= \frac{2\pi}{T} = \frac{2\pi}{1.20\,\mathrm{s}} = 5.236\,\mathrm{rad/s} \\ T &= 2\pi\sqrt{\frac{m}{k}} \implies k = m\left(\frac{2\pi}{T}\right)^2 = m\omega^2 \\ k &= (5.00\,\mathrm{kg}) \times (5.236\,\mathrm{rad/s})^2 = 137.1\,\mathrm{N/m}\end{aligned}$$

Thus, finally,

$$v_{eq} = \omega A = (5.236\,\mathrm{rad/s}) \times (0.100\,\mathrm{m}) = \boxed{0.524\,\mathrm{m/s}}$$

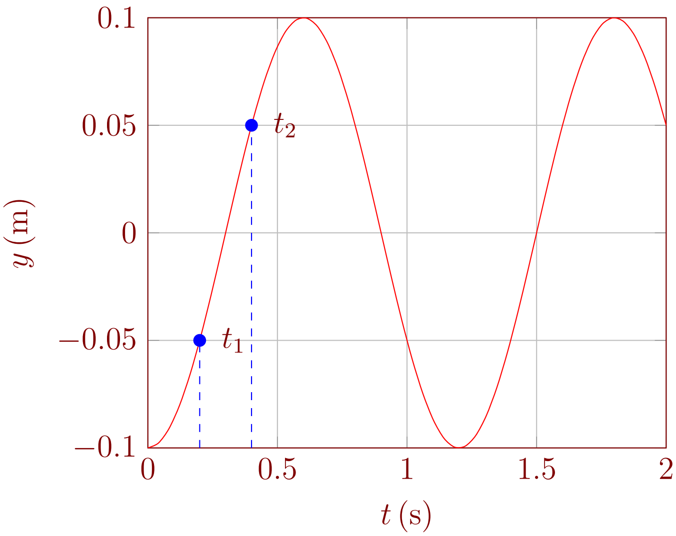
b. (10 pts.) What is its acceleration when it is 0.050 m above the equilibrium position?

Let $y_b = 0.0500\,\mathrm{m}$ equal the position. Apply Newton's second law, and use the above result for $y_R = \frac{mg}{k}$:

$$\begin{aligned}\sum F &= ma \\ Fs - mg &= ma \\ -k(y_b - y_R) - mg &= ma \\ -ky_b + ky_R - mg &= ma \\ -ky_b + k\left(\frac{mg}{k}\right) - mg &= ma \\ -ky_b &= ma \\ a &= -\frac{k}{m}y_b = -\left(\frac{137.1\,\mathrm{N/m}}{5.00\,\mathrm{kg}}\right) \times (0.0500\,\mathrm{m}) \\ a &= \boxed{-1.37\,\mathrm{m/s^2}}\end{aligned}$$

Again, this is what you would have obtained from the usual simple harmonic motion expression $a = -\frac{k}{m}y$ for motion around the equilibrium position.

c. (10 pts.) When it is moving upward, how much time is required for it to move from a point 0.0500 m below its equilibrium position to a point 0.0500 m above it?



The motion can be written as

$$y = -A\cos(\omega t)$$

where we use $-A$ because the partridge is initially pulled *down* and released from rest. The problem is then asking for the time interval between the two times t_1 and t_2 indicated on the graph.

$$\begin{aligned}
y_1 &= -A\cos(\omega t_1) \\
-\frac{y_1}{A} &= \cos(\omega t_1) \\
\arccos\left(-\frac{y_1}{A}\right) &= \omega t_1 \\
t_1 &= \frac{1}{\omega} \times \arccos\left(-\frac{y_1}{A}\right) \\
t_1 &= \frac{1}{5.236\,\text{rad/s}} \times \arccos\left(-\frac{-0.0500\,\text{m}}{0.100\,\text{m}}\right) = \boxed{0.200\,\text{s}} \\
t_2 &= \frac{1}{\omega} \times \arccos\left(-\frac{y_2}{A}\right) \\
t_2 &= \frac{1}{5.236\,\text{rad/s}} \times \arccos\left(-\frac{0.0500\,\text{m}}{0.100\,\text{m}}\right) = \boxed{0.400\,\text{s}} \\
\Delta t &= t_2 - t_1 = \boxed{0.200\,\text{s}}
\end{aligned}$$

d. (10 pts.) The motion of the partridge is stopped, and then it is removed from the spring. How much does the spring shorten?

This portion is asking for the difference between the relaxed position and the equilibrium position. The solution is above in part (a), but we can reproduce it here. In equilibrium, the sum of forces is zero.

$$\begin{aligned}
\sum F &= 0 \\
F_s - mg &= 0 \\
-k\left(y_{\text{eq}} - y_R\right) - mg &= 0 \\
\left(y_{\text{eq}} - y_R\right) &= -\frac{mg}{k} \\
\left(y_R - y_{\text{eq}}\right) &= \frac{mg}{k} = \frac{(5.00\,\text{kg}) \times (9.80\,\text{m/s}^2)}{137.1\,\text{N/m}} = \boxed{0.357\,\text{m}}
\end{aligned}$$