Physics 111: General Physics I: Mechanics and Thermodynamics
Velocity of a swinging pendulum

Problem 1: (30 pts.) A pendulum consists of a thin (massless) string of length 4.00 m, with a ball of mass 7.00 kg attached to the end. The pendulum is pulled up to an angle of 40.0° away from the vertical and released from reset. At the bottom of the pendulum's swing, what is the tension in the string?

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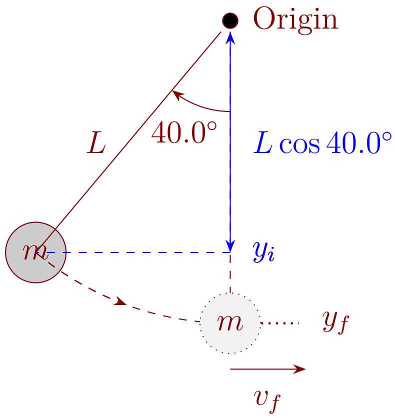

At the bottom of the swing, apply Newton's second law to find the tension F_T. Recall that for an object going in a circle of radius L, the acceleration is v^2/L.

$$\sum F = ma$$
$$F_T - mg = ma = m\frac{v_f^2}{L}$$
$$F_T = mg + m\frac{v_f^2}{L}$$

Next, find the speed v_f at the bottom.

Way # 1: Use energy conservation. Take the pivot point as the origin. The initial height is then $y_i = -L\cos 40.0°$, and the final height is $y_f = -L$, the full length of the string below the origin.

$$\begin{aligned}
E_i &= E_f \\
K_i + U_i &= K_f + U_f \\
0 + mgy_i &= \frac{1}{2}mv_f^2 + mgy_f \\
\frac{1}{2}mv_f^2 &= mg(y_i - y_f) = mg(-L\cos 40.0° - (-L)) \\
\frac{1}{2}mv_f^2 &= mgL(1 - \cos 40.0°) \\
v_f &= \sqrt{2gL(1-\cos 40.0°)} = \\
v_f &= \sqrt{2(9.80\,\text{m/s}^2) \times (4.00\,\text{m}) \times (1-\cos 40.0°)} = \boxed{4.283\,\text{m/s}}
\end{aligned}$$

Way # 2: Use simple harmonic motion. For small angles, recall that the pendulum motion is approxiately that of a simple harmonic oscillator. Although 40.0° is not obviously a "small" angle, we can still try the method to see how it works. Assume that the equation for the angle as a function of time is given by

$$\theta = A\cos(2\pi f t)$$

where A is the amplitude. To relate the angular motion to the linear velocity v, we need to express the angles in radians, not degrees:

$$A = 40.0^\circ \times \frac{2\pi\,\mathrm{rad}}{360^\circ} = 0.698\,\mathrm{rad}$$

For simple harmonic motion, the maximum speed $2\pi f A$ occurs at the equilibrium point, which is at the bottom for the pendulum. For this problem, that is an angular speed, in radians per second. There is one notational complication: For rotational motion, we use the symbol ω to represent angular velocity, but for a simple harmonic oscillator, we use the same symbol $\omega = \frac{2\pi}{T}$ for the angular frequency. To reduce confusion, we will instead write the angular velocity as the linear velocity v_f divided by the radius of circular motion, L. Thus the maximum angular speed is

$$\frac{v_f}{L} = 2\pi f A$$
$$v_f = 2\pi f A L$$

For the simple pendulum, the frequency is given by

$$f = \frac{1}{2\pi}\sqrt{g/L}$$

so the linear velocity at the bottom is

$$v_f = 2\pi f A L = 2\pi\left(\frac{1}{2\pi}\sqrt{\frac{g}{L}}\right) A L$$
$$v_f = \sqrt{gL}A = \sqrt{9.80\,\mathrm{m/s^2} \times 4.00\,\mathrm{m}} \times 0.698\,\mathrm{rad} = \boxed{4.371\,\mathrm{m/s}}$$

This value is slightly larger (2.06 %) than the correct energy value obtained using energy conservation.

Returning to the tension calculation:

$$F_T = mg + m\frac{v_f^2}{L} = m\left(g + \frac{v_f^2}{L}\right)$$
$$F_T = (7.00\,\mathrm{kg})\left(9.80\,\mathrm{m/s^2} + \frac{(4.283\,\mathrm{m/s})^2}{4.00\,\mathrm{m}}\right)$$
$$= (7.00\,\mathrm{kg})\left(9.80\,\mathrm{m/s^2} + 4.00\,\mathrm{m/s^2}\right)$$
$$= \boxed{101\,\mathrm{N}}$$