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Physics 151
Chapter 14
Damped Oscillations and Resonance

Introduction

At this point, we have developed a language to describe a simple oscillator

$$x(t) = A\cos(\omega t)$$

Now we will look at two variations on oscillating systems where the amplitude A can vary.

14.7 Damped Oscillations

We observe that real oscillations eventually are damped out, and eventually stop. That means our description of the mass/spring system is incomplete.

$$F = -kx + \quad \text{something else}$$

There can be many different types of damping. The block might be sliding on a horizontal table with friction, in which case we would have to add a constant friction force f_k opposing the motion. Or there might be some other form of damping, such as "viscous damping" or "air resistance." These are complicated phenomena, but we can often describe the main features with a simple model of the process.

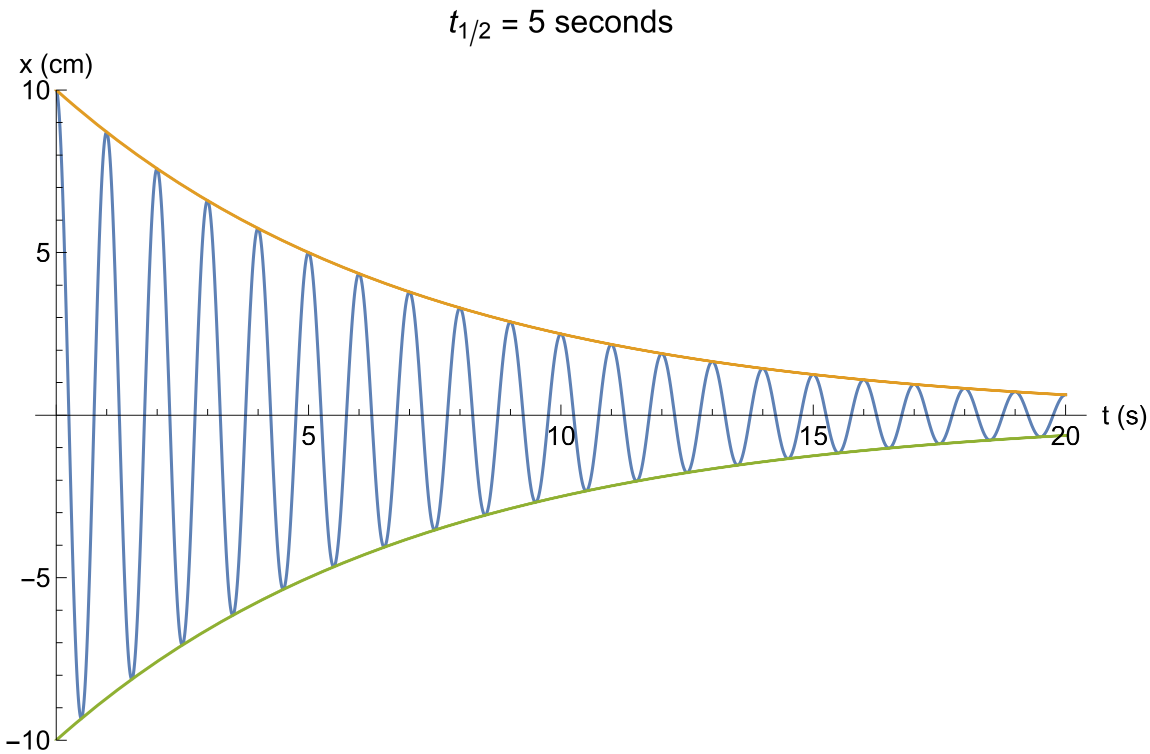
Viscous Damping

This model is useful for slow motion in a viscous fluid, magnetic breaking, electrical circuits with resistance, and a number of other situations.

$$F = -kx - bv \tag{1}$$

The constant b is known as the damping coefficient, and has units of Ns/m, or kg/s. For a typical lab experiment, where we have a mass hanging from a vertical spring, a typical value would be $b = 0.01\,\mathrm{kg/s}$.

With this damping (assuming weak damping) the oscillations gradually decrease in amplitude:



The amplitude is given by a decaying envelope

$$A(t) = A_0 e^{-\frac{bt}{2m}} = A_0 e^{-t/\tau}$$

where the time constant $\tau = 2m/b$. The time constant τ is related to the half-life $t_{1/2}$, which is the time it takes the oscillation to decay to one half of its original value, by

$$t_{1/2} = \ln(2)\,\tau \approx 0.693\tau$$

The full oscillation is then given by

$$x(t) = A(t)\cos(\omega' t) = A_0 e^{-t/\tau}\cos(\omega' t) \tag{2}$$

the oscillation frequency ω' is slightly less than the undamped mass spring frequency. Specifically,

$$\omega' = \sqrt{\frac{k}{m} - \frac{1}{\tau^2}}$$

For many applications, the difference from $\omega = \sqrt{k/m}$ is negligible.

The key feature is that the you can often think of this as simple harmonic motion, but with an amplitude A that slowly decreases with time. Hence the energy also slowly decreases with time, since $E \propto A^2$.

$$E = E_0 e^{-2t/\tau}$$

Air Drag

Another common type of damping is air drag. The magnitude of air drag is proportional to v^2, but always opposes the motion. It can be expressed as

$$F = -kx - D|v|v \tag{3}$$

where D is called the drag coefficient. If the mass is pulled to an initial position A_0 and released from rest, it will still oscillate, but with a decaying amplitude. The amplitude will decay quickly at first (when the velocity is large) but more slowly when the amplitude is small. There is no analogous analytical solution like Eq. 2.

Kinetic Friction

Finally, consider the case of a block sliding on a table with kinetic friction. The magnitude of the friction force would be a constant given by f_k, but with a direction always opposing v.

Again, if the mass is pulled to an initial position A_0 and released from rest, it will oscillate with a decaying amplitude, but there is no analogous analytical solution like Eq. 2.

14.8 Driven Oscillations and Resonance

The central idea is that if you drive a system with a oscillating force,

$$F_d = F_{\max} \cos(\omega_d t)$$

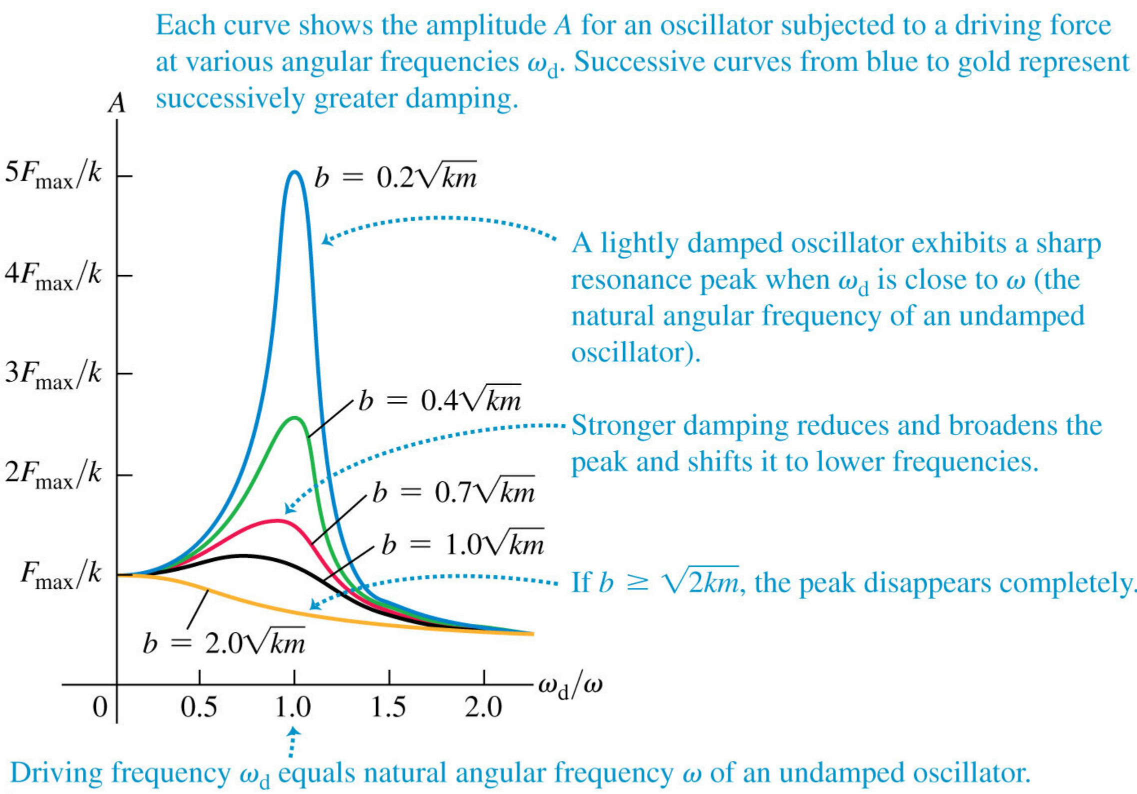
where $\omega_d = 2\pi f_d$ is the driving frequency, you will (after an initial transient) get an oscillating response

$$x(t) = A\cos(\omega_d t + \phi)$$

at the same frequency as the driving, but with a possibly different phase. If the driving frequency is far from the frequency f_0 at which the system will naturally oscillate, then you generally get a small response. But if you pick a frequency close to the natural frequency, then you will get a large response. This is known as resonance. In some case, the response near the resonant frequency can be quite dramatic.

The amplitude of that motion is given by

$$A = \frac{F_{\max}}{\sqrt{(k - m\omega_d^2)^2 + (b\omega_d)^2}}$$



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