

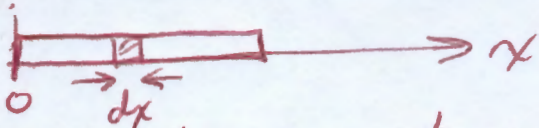
2. (20 pts.) A thin rod of length  $L$  lies along the  $x$  axis with its left end at the origin and has a *nonuniform* charge density  $\lambda = \beta x^2$ , where  $\beta$  is a positive constant.

- a. (5 pts.) The total charge on the rod is  $Q$ . What is  $\beta$ ?
- b. (15 pts.) Calculate the electric field at the origin.

2. (20 pts.) A <sup>thin</sup> rod of length  $L$  lies along the  $x$  axis with its left end at the origin and has a nonuniform charge density  $\lambda = \beta x^2$ , where  $\beta$  is a positive constant.

a. (5 pts.) The total charge on the rod is  $Q$ . What is  $\beta$ ?

b. (15 pts.) Calculate the electric field at the origin.



$$a. Q = \int_0^L \lambda dx = \int_0^L \beta x^2 dx = \frac{1}{3} \beta L^3 \Rightarrow \boxed{\beta = \frac{3Q}{L^3}}$$

$$b. E = \int_0^L \frac{1}{4\pi\epsilon_0} \frac{dq}{x^2} = \int_0^L \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{x^2} = \frac{1}{4\pi\epsilon_0} \int_0^L \frac{\beta x^2 dx}{x^2}$$

$$E = \frac{1}{4\pi\epsilon_0} \int_0^L \beta dx = \frac{1}{4\pi\epsilon_0} \beta L = \frac{1}{4\pi\epsilon_0} \frac{3Q}{L^3} L = \boxed{\frac{1}{4\pi\epsilon_0} \frac{3Q}{L^2}}$$