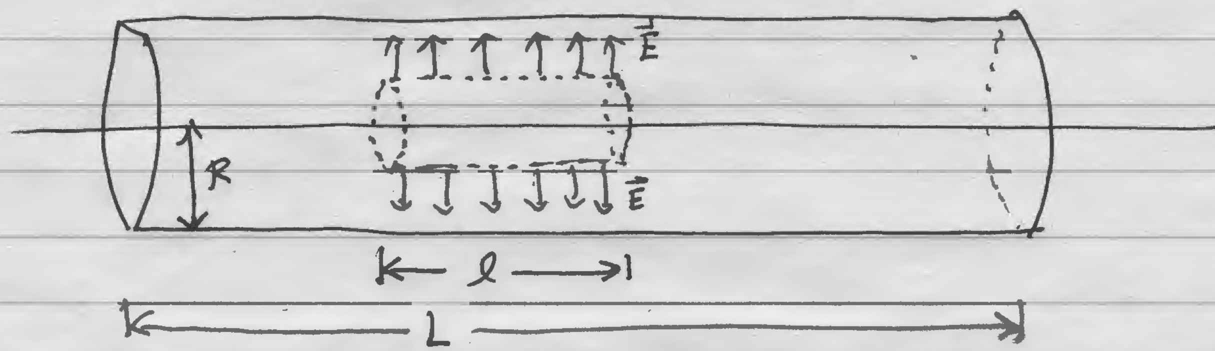
20. A cylinder of length 12 m and radius 6 cm carries a uniform volume charge density $\rho = 300\ \mathrm{nC/m^3}$. (*a*) What is the total charge of the cylinder? Find the electric field at (*b*) $r = 2$ cm, (*c*) $r = 5.9$ cm, (*d*) $r = 6.1$ cm, and (*e*) $r = 10$ cm.

Tipler ch 19 #20 Long charged cylinder.



$R = 0.06\ m$ $\qquad \rho = 300\ nC/m^3$

$L = 12\ m$ (a finite value to be concrete, but since $L \gg R$, it's effectively infinite as long as we stay away from the edges.)

(a) total charge $Q_{tot} = \rho\,(\text{Volume}) = \rho\,(\pi R^2 \cdot L)$

$$Q_{tot} = 300 \times 10^{-9} \frac{C}{m^3} \cdot \pi\,(0.06m)^2 \cdot 12m = \boxed{40.7\,nC}$$

(b) Find electric field at $r = 2\ cm = 0.02\ m$ (inside the cylinder.)

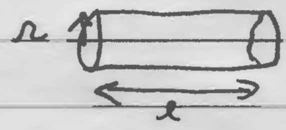
1. Sketch $\vec{E}$. It goes outward from the center of the cylinder. As long as we stay away from the edges, $\vec{E}$ is $\perp$ to the axis.

2. Design a Gaussian ~~sp~~ surface. Choose a cylinder of radius r and length ℓ, centered on the axis of the cylinder. The $\vec{E}$ flux is outward. $\vec{E}$ is normal to the curved surface. Along the flat end caps of the Gaussian surface, $\vec{E}$ is $\parallel$ to the surface, so $\vec{E} \cdot d\vec{A} = 0$ on the end caps.

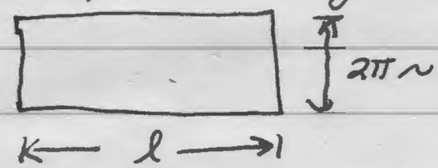
3. Compute flux through the cylindrical Gaussian surface. curved part $\Phi = \int \vec{E} \cdot d\vec{A} = \int E\,dA = E \int dA$ since E is constant on the surface.

cylinder

$\Phi = E \cdot A = E$ (surface area of curved part of cylinder)



Imagine unrolling the curved surface,



$A = 2\pi r \cdot l$

$\Phi = E\,(2\pi r l)$

Φ through flat endcaps $= 0$ since $\vec{E} \cdot d\vec{A} = 0$ there.

$\therefore \quad \Phi = E\,(2\pi r l)$

4. Compute $Q_{inside} = \rho\,(\text{volume}) = \rho\,(\pi r^2 l)$ for $r < R$.

5. Gauss's Law $\quad \Phi = Q_{inside}/\epsilon_0$

$$E(2\pi r l) = \rho(\pi r^2 l)/\epsilon_0$$

$$E = \frac{1}{2\pi r l \epsilon_0}\,\rho(\pi r^2 l) = \frac{1}{2\epsilon_0}\,\rho r$$

$$E = \frac{1}{2(8.85 \times 10^{-12}\ C^2/Nm^2)} \cdot (300 \times 10^{-9}\ C/m^3)\,(0.02\,m)$$

$$\boxed{E = 339\ N/C}$$

The remaining parts follow a similar pattern, but the Q_{inside} calculation changes once $r > R$.

(c) $r = 5.9\,cm = 0.059\,m$ (still less than R)

$$E = \frac{1}{2\epsilon_0}\rho r = \boxed{1000\ N/C}$$

(d) $r = 6.1\,cm = 0.061\,m$ (now more than R)

Return to Gauss's Law $\Phi = Q_{inside}/\epsilon_0$ — note R here.

$$E(2\pi r l) = \rho(\pi R^2 l)/\epsilon_0$$

$$E = \frac{1}{2\pi r l}\,\frac{\rho \pi R^2 l}{\epsilon_0} = \frac{1}{2\pi\epsilon_0}\,\frac{\rho \pi R^2}{r} = \frac{1}{2\epsilon_0}\,\frac{\rho R^2}{r}$$

$$E = \frac{1}{2(8.85 \times 10^{-12}\ C^2/Nm^2)}\,\frac{(300 \times 10^{-9}\ C/m^3)(0.06\,m)^2}{(0.061\,m)} = \boxed{1000\ N/C}$$

cylinder

(e) $r = 10\,cm = 0.10\,m$

$$E = \frac{1}{2\pi\epsilon_0}\frac{\rho R^2}{r}$$

$$E = \frac{1}{2(8.85\times10^{-12}\,C^2/Nm^2)}\frac{(300\times10^{-9}\,C/m^3)(0.06\,m)^2}{(0.10\,m)}$$

$$\boxed{E = 610\,N/C}$$