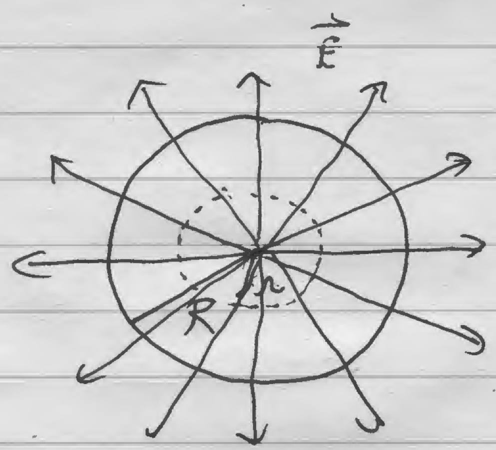
18. A sphere of radius 6 cm carries a uniform volume charge density $\rho = 450\ \text{nC/m}^3$. (*a*) What is the total charge of the sphere? Find the electric field at (*b*) $r = 2$ cm, (*c*) $r = 5.9$ cm, (*d*) $r = 6.1$ cm, and (*e*) $r = 10$ cm.

Tipler ch 19 #18. Charged sphere.



$R = 0.06\ m$

$\rho = 450 \times 10^{-9}\ C/m^3$

a) $Q_{tot} = \rho\,(\text{volume}) = \rho\left(\frac{4}{3}\pi R^3\right)$

$Q_{tot} = 4.07 \times 10^{-10}\ C$

b) Find E at $r = 0.02\ m$ (inside R).

1. Sketch $\vec{E}$. It goes radially outward.
2. Design a Gaussian surface. Choose a sphere of radius r centered on center of sphere. $\vec{E}$ will always be parallel to $d\vec{A}$.
3. Compute flux

   $\Phi = \oint \vec{E} \cdot d\vec{A} = \oint E\,dA$ since dot product = 1

   $= E \oint dA$ since E is same magnitude everywhere on the surface of a sphere.

   $\Phi = E\,4\pi r^2$
4. Compute $Q_{inside} = \rho\left(\frac{4}{3}\pi r^3\right)$ as long as $r \le R$

   (note not R.)

   $Q_{inside} = \frac{4}{3}\pi (0.02\,m)^3 (450 \times 10^{-9}\ C/m^3) = 1.51 \times 10^{-11}\ C$
5. Gauss's Law

   $\Phi = Q_{inside}/\epsilon_0$. Spelling it out symbolically,

   $E\,(4\pi r^2) = \frac{4}{3}\pi r^3 \rho / \epsilon_0$

   $E = \frac{1}{4\pi\epsilon_0}\,\frac{4}{3}\pi r \rho = \boxed{339\ N/C}$

The remaining parts follow a similar structure, but the Q_{inside} calculation changes once $r > R$.

(c) $r = 0.059$ m (still inside)

$$E = \frac{1}{4\pi\epsilon_0}\frac{4}{3}\pi r \rho = \boxed{1000\ N/C}$$

(d) $r = 0.061$ m outside the sphere! Go back to

$$\Phi = Q_{inside}/\epsilon_0$$

$E\,4\pi r^2 = Q_{inside}/\epsilon_0$, but now $Q_{inside} = Q_{tot}$ = all of the charge $= 4.07 \times 10^{-10}$ C from part (a)

$$\therefore E = \frac{1}{4\pi\epsilon_0}\frac{Q_{tot}}{r^2} = \left(8.99 \times 10^9 \frac{Nm^2}{C^2}\right)\frac{4.07 \times 10^{-10}\,C}{(0.061m)^2}$$

$$\boxed{E = 983\ N/C}$$

(e) $r = 0.10$ m. Outside R. Use the same formula as in (d)

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q_{tot}}{r^2} = \left(8.99 \times 10^9 \frac{Nm^2}{C^2}\right)\frac{4.07 \times 10^{-10}\,C}{(0.10m)^2}$$

$$\boxed{E = 366\ N/C}$$

Graphing