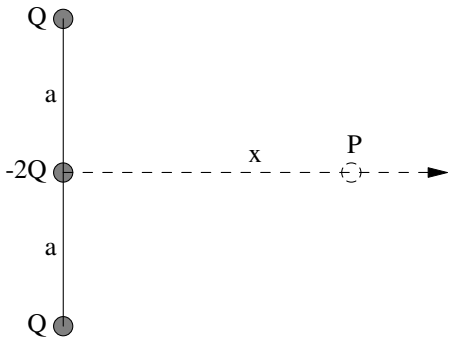
Problem 3: (20 pts.) Consider the arrangement of charges (known as a quadrupole) shown in the figure.



a. (10 pts.) What is the potential V at point P a distance x along the x axis?

b. (10 pts.) If P is far from the origin (i.e. $x >> a$) it is possible to show (but you should *not* do it on this test) that

$$V \approx -\frac{1}{4\pi\epsilon_0}\frac{Qa^2}{x^3} \ .$$

Use this expression for V to compute the x-component of the electric field at point P. Give a brief qualitative explanation of why you should expect the sign you get.

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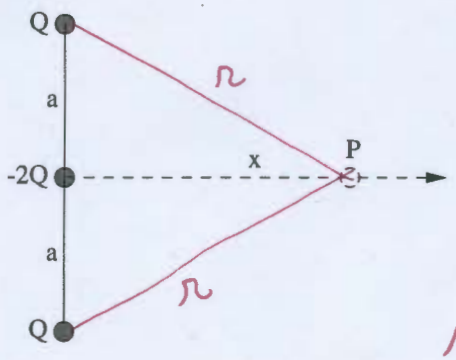

$r = \sqrt{x^2 + a^2}$

a. (10 pts.) What is the potential V at point P a distance x along the x axis?

$$V = \frac{2kQ}{r} + \frac{k(-2Q)}{x} = 2kQ\left(-\frac{1}{x} + \frac{1}{\sqrt{x^2+a^2}}\right)$$

b. (10 pts.) If P is far from the origin (i.e. $x >> a$) it is possible to show (but you should *not* do it on this test) that

$$V \approx -\frac{1}{4\pi\epsilon_0}\frac{Qa^2}{x^3}.$$

Use this expression for V to compute the x-component of the electric field at point P. Give a brief qualitative explanation of why you should expect the sign you get.

$$E = -\frac{dV}{dx} = -\frac{d}{dx}\left(-\frac{1}{4\pi\epsilon_0}\frac{Qa^2}{x^3}\right) = \boxed{-\frac{3Qa^2}{4\pi\epsilon_0 x^4}}$$

The $-2Q$ charge is slightly closer, so the electric field points towards the $-2Q$. In addition, the $+Q$ charges are less effective, since the vertical components of the electric fields cancel.