Problem 6: (40 pts.) A parallel plate capacitor is made from two circular plates of radius 0.050 m separated by 0.001 m. The voltage across the plates is given as a function of time by

$$V(t) = V_0 + V_1 \frac{t}{\tau} \ ,$$

where $V_0 = 30$ Volts, $V_1 = 1000$ Volts, and $\tau = 2$ s.

a. (30 pts.) At a time of $t = 5$ s, what is the magnetic field strength inside the capacitor at a distance of 0.030 m from the central axis?

b. (10 pts.) At that same time of $t = 5$ s, what is the magnetic field strength outside the capacitor at a distance of 0.080 m from the central axis?

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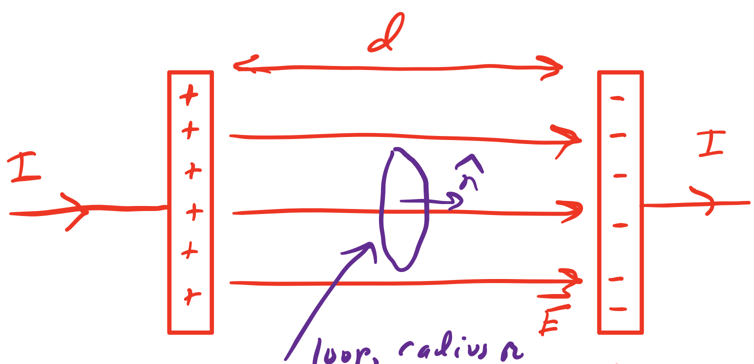
(a)



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \left[I_{enc} + \epsilon_0 \frac{d}{dt}\Phi_E \right] , \quad I_{enc} = 0$$

$$\Phi_E = \int \vec{E}\cdot\hat{n}\, dA = \int E\, dA = E\int dA = EA = \left(\frac{\Delta V}{d}\right)\pi r^2$$

$$\frac{d\Phi_E}{dt} = \frac{\pi r^2}{d}\cdot\frac{d}{dt}(\Delta V) = \frac{\pi r^2}{d}\frac{V_1}{\tau} , \quad r = 0.03\,m, \ d = 0.08\,m$$

$$\oint \vec{B}\cdot d\vec{l} = B\cdot 2\pi r$$

$$\therefore \ 2\pi r B = \mu_0 \epsilon_0 \frac{\pi r^2}{d}\frac{V_1}{\tau}$$

$$B = \frac{\mu_0 \epsilon_0 \pi r^2 V_1}{2\pi r\, d\, \tau} = \frac{\mu_0 \epsilon_0 r V_1}{2 d \tau} =$$

$$\boxed{B = 8.34 \times 10^{-14}\, T.}$$

Tiny!

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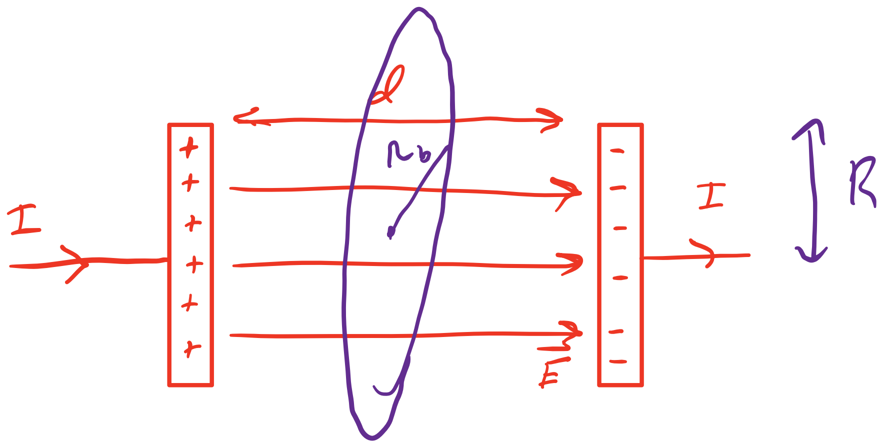
(b)



(b) For a larger distance, $r_b = 0.08$ m, the flux is confined to the area between the plates where $E \neq 0$, so $\Phi_E = \left(\frac{\Delta V}{d}\right) \pi R^2$, where $R = 0.05$ m. The line integral $\oint \vec{B} \cdot d\vec{l}$ is still $B\, 2\pi r_B$, where r_b is where you want to measure the field.

$$\therefore \quad \oint \vec{B} \cdot d\vec{l} = \mu_0 \left[I + \epsilon_0 \frac{d\Phi_E}{dt} \right]$$

gives

$$2\pi r_b B = \mu_0 \epsilon_0 \frac{\pi R^2}{d} \frac{V_1}{\tau}$$

$$B = \frac{\mu_0 \epsilon_0 R^2}{2 r_b d} \frac{V_1}{\tau} = \boxed{1.74 \times 10^{-15}\ \text{T}}$$