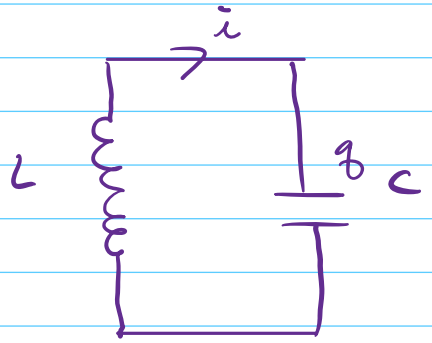
30.33 • In an L-C circuit, $L = 85.0$ mH and $C = 3.20\ \mu$F. During the oscillations the maximum current in the inductor is 0.850 mA. (a) What is the maximum charge on the capacitor? (b) What is the magnitude of the charge on the capacitor at an instant when the current in the inductor has magnitude 0.500 mA?



$L = 85.0\ \text{mH}$

$C = 3.20\ \mu\text{F}$

$i_{max} = 0.850\ \text{mA}$

(a) Max energy stored in inductor $= U_{max} = \frac{1}{2} L i_{max}^2$

Max energy stored in capacitor $= U_{max} = \frac{1}{2} q_{max} V_{max}$

but $q = CV$, so $V = q/C$

$$U_{max} = \frac{1}{2} \frac{q_{max}^2}{C}$$

Conservation of energy $\Rightarrow U_{max} = U_{max_2}$

$$\frac{1}{2} L i_{max}^2 = \frac{1}{2} \frac{q_{max}}{C}$$

$$q_{max} = \sqrt{LC}\, i_{max}$$

$$\sqrt{LC} = \sqrt{(85 \times 10^{-3}\ \text{H})(3.20 \times 10^{-6}\ \text{F})} = 5.215 \times 10^{-4}\ \Omega$$

$$q_{max} = (5.215 \times 10^{-4}\ \Omega)\left(0.850 \times 10^{-3}\ \frac{\text{C}}{\Omega}\right) =$$

$$\boxed{q_{max} = 4.43 \times 10^{-7}\ \text{C}}$$

(b) when $i = 0.500\ \text{mA}$, $q = ?$

Again, use conservation of energy

$$U_{total} = U_L + U_C$$

but $U_{total} = \frac{1}{2} L i_{max}^2$

$\therefore \frac{1}{2} L i_{max}^2 = \frac{1}{2} L i^2 + \frac{1}{2} \frac{q^2}{C}$.

$q^2 = LC\left(i_{max}^2 - i^2\right)$

$q = \sqrt{LC}\sqrt{i_{max}^2 - i^2}$

$q = (5.215 \times 10^{-4}\,\Omega)\left(\sqrt{(0.85 \times 10^{-3}\,A)^2 - (0.50 \times 10^{-3}\,A)^2}\right)$

$\boxed{q = 3.58 \times 10^{-7}\,C}$