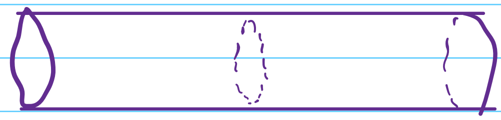
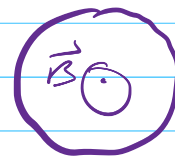
29.47 •• **CALC** A very long, straight solenoid with a cross-sectional area of $2.00\ \text{cm}^2$ is wound with 90.0 turns of wire per centimeter. Starting at $t = 0$, the current in the solenoid is increasing according to $i(t) = (0.160\ \text{A/s}^2)t^2$. A secondary winding of 5 turns encircles the solenoid at its center, such that the secondary winding has the same cross-sectional area as the solenoid. What is the magnitude of the emf induced in the secondary winding at the instant that the current in the solenoid is 3.20 A?

End View





B due to solenoid $= \mu_0 n i$.

Φ = flux through coil $= N B A$

($N = 5$, $B = \mu_0 n i$, $A = 2.00\ \text{cm}$)

$A = 2.00\ \text{cm}^2 \times \left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2 = 2 \times 10^{-4}\ \text{m}^2$

$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(N A \mu_0 n i)$

$\mathcal{E} = N A \mu_0 n \left(-\frac{di}{dt}\right)$

$i = 0.160\ \frac{A}{s^2} \cdot t^2$

$\frac{di}{dt} = 0.320\ \frac{A}{s^2} \cdot t$. at what time?

Given $3.20\ A = 0.160\ \frac{A}{s^2}\ t^2$

$t = \sqrt{\frac{3.20}{0.160}}\ s = \sqrt{20}\ s$

Plugging in . . .

$$\mathcal{E} = -NA\mu_0 n \left(\frac{di}{dt}\right)$$

$$\mathcal{E} = -(5)(2\times10^{-4}\,m^2)(4\pi\times10^{-7}\,Tm/A)\cdot$$

$$\left(\frac{90\ \text{turns}}{cm}\times\frac{100\,cm}{m}\right)\times\left(0.320\,\frac{A}{s^2}\right)\sqrt{20s}$$

$$|\mathcal{E}| = 1.62\times10^{-5}\,V = 16.2\,\mu V$$