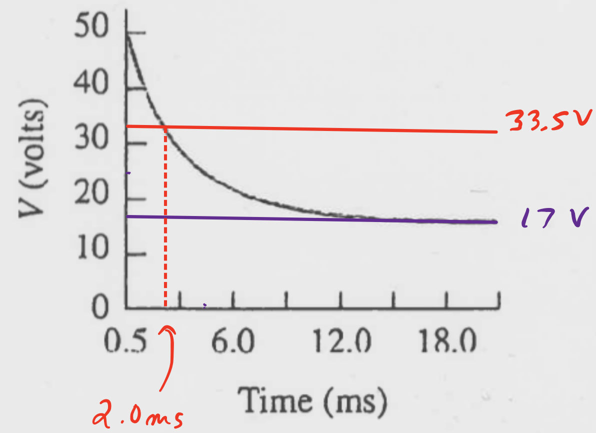
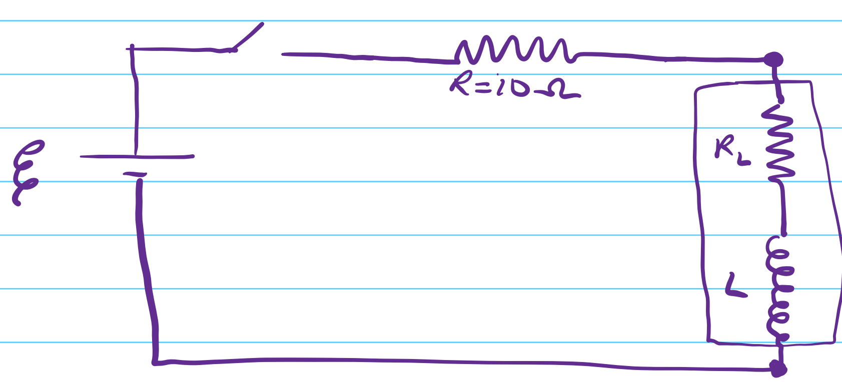
30.61. In the lab, you are trying to find the inductance and internal resistance of a solenoid. You place it in series with a battery of negligible internal resistance, a 10.0-Ω resistor, and a switch. You then put an oscilloscope across one of these circuit elements to measure the voltage across that circuit element as a function of time. You close the switch, and the oscilloscope shows voltage versus time as shown in Fig. 30.21. (a) Across which circuit element (solenoid or resistor) is the oscilloscope connected? How do you know this? (b) Why doesn't the graph approach zero as $t \to \infty$? (c) What is the emf of the battery? (d) Find the maximum current in the circuit. (e) What are the internal resistance and self-inductance of the solenoid?

Figure 30.21 Problem 30.61.

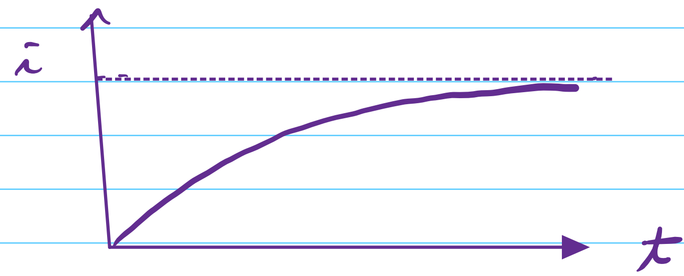


Real inductors have resistance, so the actual device can be modeled as a pure inductor plus a pure resistor

Thus our circuit looks like this:



Think about the current: There is no "instant on."
Instead, the current builds up more gradually.



(a) Since $v_R = iR$, it will increase in time.
Since $v_L = -L\dfrac{di}{dt}$, it will decrease in time.

$\therefore$ The graph in the problem shows v_L, the voltage across the inductor.

(b) as $t \to \infty$, i reaches its steady state value. Since the V_L is not zero, there must be some resistance in the inductor.

(c) Originally, at $t = 0$, the current is 0 so the voltage across the inductor = the battery. (The voltage across the resistor is 0.)

$\therefore$

$$\mathcal{E} = 50\,V.$$

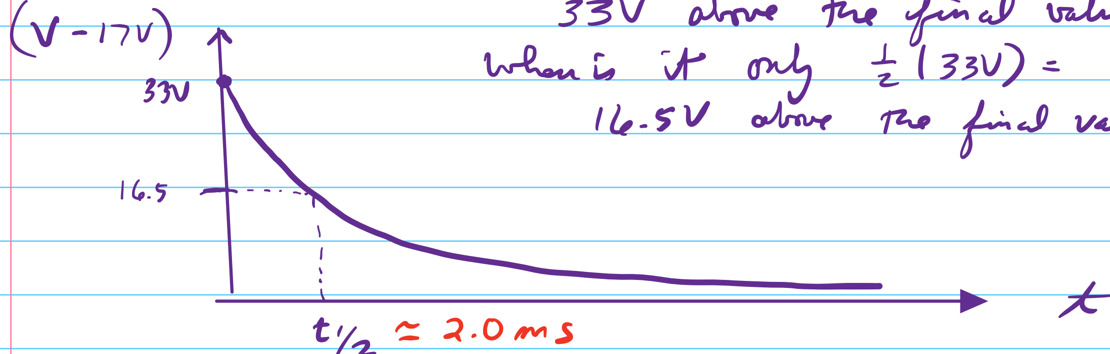
(d) The maximum current is at large time. At large t, the voltage across the inductor = 17 V (estimating from the graph) so $V_R = 33V = i_{max} R$

$$\therefore \quad i_{max} = \frac{33V}{10\,\Omega} = 3.3A.$$

(e) The internal resistance of the inductor is

$$R_L = \frac{17V}{3A} \simeq 5.67\,\Omega.$$

The inductance L can be estimated from the half-time on the graph: It starts at 33V above the final value. When is it only $\frac{1}{2}(33V) =$ 16.5V above the final value?



A rough estimate of $t_{1/2} = 2.0\,ms$.

$\tau = ?$ Solve: $\frac{1}{2} = e^{-t_{1/2}/\tau}$

$$\ln(2) = t_{1/2}/\tau$$

$$\tau = \frac{t_{1/2}}{\ln(2)} = \frac{2.0\,ms}{0.693} = 2.89\,ms$$

but $\tau = \frac{L}{R_{total}} \Rightarrow L = \tau \cdot R_{total}$

$$L = (2.89 \times 10^{-3}\,s)(15.67\,\Omega) = 0.045\,H$$

$$\boxed{L = 0.045\,H = 45\,mH}$$