Problem 6: (30 pts.) A 200 kW radio station is broadcasting radio waves at a linear frequency of 99.0 MHz. Assume that the waves propagate outward equally in all directions.

a. (10 pts.) At a distance of 2200 m away from the broadcast tower, what is the maximum amplitude of the magnetic field in the radio wave?

b. (20 pts.) Suppose you wish to receive these signals by using a circular loop of radius 0.075 m as an antenna. You position the loop so that the magnetic field of the radio waves is perpendicular to the plane of the loop. (You may assume the wave is polarized so that the magnetic field oscillates in a single plane. Since the loop is much smaller than the wavelength of the wave, you may also assume that at any instant, the magnetic field has the same value everywhere on the loop.) What is the maximum value of the induced emf in the loop? You may find it helpful to assume that the "x" value in the travelling wave forumla is 0 at the location of the loop.

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$$S_{av} = \frac{P}{4\pi r^2} = \frac{200{,}000\ W}{4\pi(2200)^2} = 3.288 \times 10^{-3}\ W/m^2$$

$$S_{av} = \frac{1}{2\mu_0} EB = \frac{1}{2\mu_0}(Bc)B = \frac{c}{2\mu_0}B^2 \Rightarrow B = \sqrt{\frac{2\mu_0 S_{av}}{c}}$$

$$B = 5.249 \times 10^{-9}\ T$$

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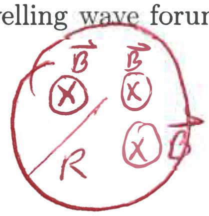

$$\mathcal{E} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A} = -\frac{d}{dt}(BA) = -\left(\frac{dB}{dt}\right)\pi R^2$$

$B = B_m \sin(kx - \omega t) = B_m \sin(-\omega t)$, using the hint.

$\omega = 2\pi f$.

$$\mathcal{E} = \omega B_m \underbrace{\cos(\omega t)}_{\text{max value is } 1} \cdot \pi R^2$$

$$\mathcal{E}_{max} = (2\pi f) B_m \pi R^2 = 2\pi(99.0 \times 10^6)(5.249 \times 10^{-9})\pi(0.075)^2$$

$$\boxed{\mathcal{E}_{max} = 0.0577\ \text{Volts}}$$