Complex Numbers

$e^{i\theta} = \cos\theta + i\sin\theta \qquad \cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2} \qquad \sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i} \qquad Z_R = R \qquad Z_C = \frac{1}{i\omega C}$
$Z_L = i\omega L$

Expansions

$\cos(x) \approx 1 - \frac{1}{2!}x^2 + \ldots \qquad \sin(x) \approx x - \frac{1}{3!}x^3 + \ldots \qquad e^x \approx 1 + x + \frac{1}{2!}x^2 + \ldots$
$(1+x)^n \approx 1 + nx + \ldots$

Damped Harmonic Oscillator

$m\ddot{x} = -kx - b\dot{x} \qquad \omega_0 = \sqrt{k/m} \qquad \gamma = b/m \qquad \omega_v = \omega_0\sqrt{1 - \frac{\gamma^2}{4\omega_0^2}} \qquad A(t) = A_0 e^{-\gamma t/2}$
$Q = \frac{\omega_0}{\gamma} \qquad \tau = \frac{1}{\gamma} \qquad Re = \frac{\rho v L}{\mu}$

Forced Vibrations and Resonance

$m\ddot{x} = -kx - b\dot{x} + F_0\cos(\omega t) \qquad x(t) = A\cos(\omega t - \delta) \qquad A(\omega) = \frac{F_0/m}{\sqrt{(\omega_0^2-\omega^2)^2 + (\gamma\omega)^2}}$
$\tan\delta(\omega) = \frac{\gamma\omega}{(\omega_0^2-\omega^2)} \qquad \omega_m = \omega_0\sqrt{1 - \frac{1}{2Q^2}} \qquad \bar{P}(\omega) = \frac{F_0^2\omega_0}{2kQ}\frac{1}{\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)^2 + \frac{1}{Q^2}}$

Coupled Oscillators

$\omega_0 = \sqrt{\frac{T}{ma}} \qquad \omega_n = 2\omega_0 \sin\left[\frac{n\pi}{2(N+1)}\right] \qquad A_{jn} = C_n \sin\left(\frac{jn\pi}{N+1}\right)$

Fourier Series

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos k_n x + b_n \sin k_n x\right] \qquad k_n = n\frac{2\pi}{\lambda}$
$a_n = \frac{2}{\lambda}\int_0^{\lambda}\cos k_n x f(x)dx \qquad b_n = \frac{2}{\lambda}\int_0^{\lambda}\sin k_n x f(x)dx$

Some Fourier Series

Square Wave

The Fourier Series for a square wave of height $\pm h$ and period λ is

$$square(x) = \frac{4h}{\pi}\left[\sin\left(\frac{2\pi x}{\lambda}\right) + \frac{1}{3}\sin\left(3\times\frac{2\pi x}{\lambda}\right) + \frac{1}{5}\sin\left(5\times\frac{2\pi x}{\lambda}\right)\ldots\right]$$

Triangle Wave

The Fourier Series for a triangle wave of height $\pm h$ and period λ is

$$triangle(x) = \frac{8h}{\pi^2}\left[\sin\left(\frac{2\pi x}{\lambda}\right) - \frac{1}{3^2}\sin\left(3\times\frac{2\pi x}{\lambda}\right) + \frac{1}{5^2}\sin\left(5\times\frac{2\pi x}{\lambda}\right)\ldots\right]$$

Sawtooth Wave

The Fourier Series for a sawtooth wave of height $\pm h$ and period λ is

$$saw(x) = \frac{2h}{\pi}\left[\sin\left(\frac{2\pi x}{\lambda}\right) - \frac{1}{2}\sin\left(2\times\frac{2\pi x}{\lambda}\right) + \frac{1}{3}\sin\left(3\times\frac{2\pi x}{\lambda}\right) - \frac{1}{4}\sin\left(4\times\frac{2\pi x}{\lambda}\right)\ldots\right]$$