Physics 238

Alternating Current Circuits and Filters I
Due Wednesday, April 23, 2025, 1:15 pm

1 INTRODUCTION

Many of the ideas we discuss in the context of mechanical oscillations carry over into the realm of oscillations in electrical circuits. Broadly speaking, the process of analyzing time-varying signals is known as “signal processing”.

In this experiment you will examine the effects of resistors and capacitors on alternating current signals. In previous experiments, we have primarily focused on studying how signals vary in time. In this experiment, we shift our focus to thinking about signals in the frequency domain. Specifically, you will construct high pass, low pass, and bandpass filters. These circuits will have a response that depends on the frequency of the incoming signal.

1.1 Notation Conventions

We will typically use lower case letters, e.g. v, for time-varying quantities, and upper case letters, e.g. V, for constant quantities. Of course, sometimes those “constant” quantities themselves will be slowly varying in time, so the distinction is not absolute. Typically, we will consider an input signal of the form

$$v_{\text{in}}(t) = V_{\text{in}} \cos \omega t \,,$$

and the corresponding output signal will be of the form

$$v_{\text{out}}(t) = V_{\text{out}} \cos(\omega t + \phi) \,.$$

The constants V_{in} and V_{out} are known as amplitudes. The ratio $A = V_{\text{out}}/V_{\text{in}}$ is known as the voltage gain. Even though A is less than 1 in the absence of amplification of some sort, it is still called the voltage “gain”.

1.2 Complex Impedance

Much of the analysis is streamlined if you use complex impedance. The basis is Euler’s identity:

$$e^{j\phi} = \cos \phi + j \sin \phi$$

where we will use $j = \sqrt{-1}$ for the imaginary number to avoid confusion with the electrical current i. This leads to two different ways to represent a complex number:

$$a + jb = Ae^{j\phi}$$
$$A = \sqrt{a^2 + b^2}$$
$$\tan\phi = \frac{b}{a}$$

where a, b, A, and ϕ are all real.

We can analyze AC circuits by using a complex impedance Z in lieu of the usual plain resistance R from DC circuits. The relevant values for different components are:

Device	**Impedance** (Ω)
Resistor	R
Inductor	$Z_L = j\omega L$
Capacitor	$Z_C = \frac{1}{j\omega C}$

Then if a device is subject to an oscillating voltage

$$v(t) = Ve^{j\omega t}$$

the current is related to that voltage by $v = iZ$, where Z is the complex impedance. Note that since Z is complex, it can affect both the amplitude and the phase of the current.

2 RC High Pass Filter

In this part, you will study the voltage gain and phase shift of a simple RC circuit as a function of frequency.

2.1 Theory

Consider the circuit shown in Fig. 1.

The total impedance of the circuit is $Z = R + Z_C$, and the current is $i = v_{\text{in}}/Z$. The output voltage, measured across the resistor, is iR. Putting it all together, The amplitude of the output voltage is then $V_{\text{out}} = V_{\text{in}}\frac{R}{|Z|}$, where the $|Z|$ is the magnitude of the total impedance, given by

$$|Z| = \sqrt{R^2 + \frac{1}{(\omega C)^2}}\ .$$

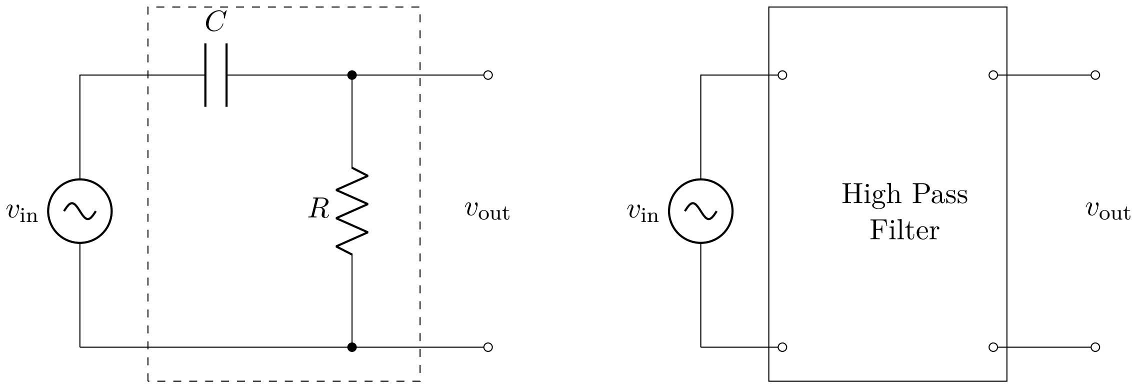

Figure 1: RC High Pass Filter. The region enclosed in the dotted box is a high-pass filter, and we sometimes represent it simply as a black box with inputs and outputs.

The voltage gain $A(\omega)$ of the circuit is defined to be V_{out}/V_{in}, and is given for this circuit by

$$A(\omega) = \frac{V_{out}}{V_{in}} = \frac{R}{Z} = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}} \;, \tag{1}$$

where the last expression follows from some algebra. At small ω the gain is near 0, but as the frequency increases, the gain rises and approaches 1. Hence this circuit is known as a high-pass filter—high frequency signals are passed through, but low frequency signals are blocked.

The "breakpoint" frequency is defined to be $\omega_b = 1/(RC)$. At this frequency, $A = 1/\sqrt{2}$, and the power has decreased to 1/2 of its peak value.

The phase ϕ of the output voltage also varies with frequency, and is given by

$$\phi(\omega) = \tan^{-1}\left(\frac{1}{\omega RC}\right) \tag{2}$$

2.2 Setup

Construct the circuit shown in Fig. 1. Use the Hi-Ω output of the function generator. (This output has an impedance of 600 Ω.) Use $R = 3.30\,\mathrm{k\Omega}$ and $C = 0.022\,\mathrm{\mu F}$.[1] These are nominal values only. *Measure, record, and* use *the actual resistance and capacitance values.* Do not include the 600 Ω output impedance of the function generator in the total resistance R_{total} of your circuit.

[1] The capacitor may be labeled "223" for $22 \times 10^3\,\mathrm{pF} = 22\,\mathrm{nF} = 0.022\,\mathrm{\mu F}$.

Calculate the expected linear breakpoint frequency $f_b = \omega_b/(2\pi) = 1/(2\pi R_{\text{total}} C)$ for your circuit.

Set the function generator to sine waves.

Set up the oscilloscope for dual trace mode. Monitor the input voltage on Channel 1, and the output voltage (The voltage across R) on Channel 2. Use the "DC" settings for both channels. (The "AC" setting is effectively just a high-pass filter that subtracts out the average value of the incoming signal.)

2.3 Voltage Gain and Phase Difference

Use the `Measure` feature to measure the voltage gain. Note that since you are taking a ratio $V_{\text{out}}/V_{\text{in}}$, it doesn't matter if you use the "peak", "peak-to-peak", or "RMS" values. The `Cyc RMS` value averages over a whole cycle of oscillation, so it is usually the most robust measurement. *Adjust the horizontal and vertical scales so that a few full oscillations nearly fill the screen.*

To measure the phase difference, use the `Cursor` menu. (Note that if you use `Cursor 1` and `Cursor 2` to mark the horizontal positions of two peaks, the oscilloscope can calculate the time difference Δt for you. The phase is then given by

$$\phi = 2\pi \frac{\Delta t}{T} = 2\pi f(\Delta t)$$

where $T = 1/f$ is the oscillation period. For this lab, don't worry about the sign of Δt.

2.3.1 Data Acquisition

Take the data necessary to measure the voltage gain and phase difference over a wide range of frequencies, going from well below f_b (*e.g.* $f_b/10$) to well above f_b (*e.g.* $10 f_b$). Record your *raw data* in a clear table. Note that the *raw data* is the data you get directly from the oscilloscope, not the results of calculations you perform on that data.

Note that as you change the frequency, you should change both the horizontal and vertical scales, as necessary, to get reliable readings.

You don't need a lot of data points. Take a few data points near the ends of the range, and more near the breakpoint frequency. If you *plot your data as you go along*, it should become clear when you have enough data. Don't waste time taking lots of closely-spaced data points.

2.3.2 Analysis

From your raw data, calculate the voltage gain A and phase shift ϕ as a function of frequency.

Fit Eq. 1 to your data. Note that you can only fit the product RC, you can't determine the individual values from a fit. Make a plot showing your experimental data along with the fit. Discuss the fit and fit parameters.

If everything agrees well, there is usually little more to say. If there are significant discrepancies, try to resolve them, perhaps by going back and re-measuring some points, if necessary. Comment on both the scale of any errors as well as any systematic trends in errors. You may well find that the uncertainties in your component values (as measured by the DMM) are the dominant uncertainties in this experiment.

Similarly, fit Eq. 2 to your data, and present and discuss your results.

3 RC Low Pass Filter

3.1 Setup

Interchange the position of the resistor and capacitor in Fig. 1. Measure the output voltage across the capacitor.[2]

3.2 Theory

The theoretical gain for this circuit is

$$A = \frac{V_{out}}{V_{in}} = \frac{X_c}{Z} = \frac{1}{\sqrt{1 + (\omega RC)^2}} \ . \tag{3}$$

The phase shift is

$$\tan\phi = -\omega RC \tag{4}$$

The breakpoint frequency is the same as before.

3.3 Voltage Gain and Phase Difference

3.3.1 Data Acquisition

Measure the voltage gain over a wide range of frequencies, going from well below f_b to well above f_b. Also, make a qualitative note of how the phase difference ϕ varies across

[2] You may be tempted to simply move the oscilloscope probes to measure the voltage across the capacitor, but this won't work because of the way the oscilloscope probes are grounded. The negative terminals of the oscilloscope probes are connected internally to the oscilloscope ground, which is also connected to the third prong on the power cord. Thus the negative terminals of both probes must always be connected to the same spot in the circuit. In fact, you don't even need to connect the negative terminal of the second probe, though it's good practice to do so.

frequencies. (This is simply a rough estimate of the phase difference as "large" or "small" compared to the period. No quantitative measure of the phase difference is required.)

3.4 Analysis

Fit Eq. 3 to your data. Make a plot showing your experimental data along with the fit. Discuss the fit and fit parameters.

Discuss your qualitative observations for the phase shift and compare them to the expectations based on Eq. 4.

4 Bandpass Filter

4.1 Setup

A "bandpass" filter is a filter that passes only a narrow range (or "band") of frequencies. Signals with frequencies either higher or lower than that band are blocked. One simple way to build a bandpass filter is to start with a low-pass filter (which blocks high frequencies) and follow it with a high-pass filter (which blocks low frequencies).

Design and build a bandpass filter by combining low-pass and high-pass filters. Start by using your low-pass filter from the previous part. This will tend to pass all signals with frequencies lower than f_{lowpass}. Use the *output* of your low-pass filter as the *input* to a high pass filter. Your high-pass filter should have a breakpoint frequency $f_{highpass}$ slightly *lower* than f_{lowpass}. That is, the combination of filters will tend to pass through only frequencies f in the range

$$f_{\text{highpass}} < f < f_{\text{lowpass}} . \tag{5}$$

Design the second stage of the filter to have a higher impedance to prevent it from "loading down" the first stage. This is easiest to do by simply changing R (either by replacing your resistor or adding a second resistor to the circuit.) *Include a circuit diagram of your bandpass filter in your lab writeup. Give the values for all the components, and show where you measured the output.*

4.2 Voltage Gain and Phase Difference

4.2.1 Data Acquisition

Measure the voltage gain and note the qualitative phase difference for your circuit as a function of frequency over a vary wide range of frequencies. Again, don't waste time taking too many closely-spaced data points. You are looking for the general overall behavior over a very wide range of frequencies. **If you *plot your data as you go along*, it should**

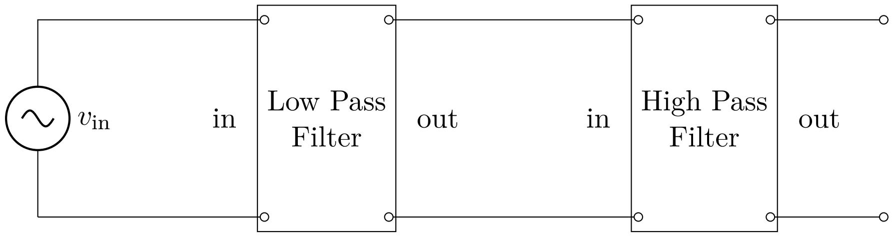

Figure 2: Block diagram of a bandpass filter, consisting of a low-pass filter followed by a high-pass filter.

become clear when you have enough data. See the Analysis section below for further details.

4.3 Analysis

Plot $\log(A)$ as a function of $\log(f)$. (The log-log plot is a useful way to show a wide range of data on a single plot.) Include on your graph the theoretical prediction for each stage of the filter operating independently. (You don't need to compute a combined theoretical response.) Discuss your results.

Discuss qualitatively how the phase varied with frequency.

With more complex circuits, it is possible to get a much sharper frequency response, but this simple circuit illustrates the basic features of a bandpass filter.